System modeling and simulation(ME340)

Chapter 5. state-variable models and simulation methods

5.1 state-variable models

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Different forms of dynamic models

Dynamic models derived from basic physical principles can appear in several forms:

- 1. As a single equation (which is called the *reduced form),*
- 2. As a set of coupled first-order equations (which is called the *Cauchy or state-variable* form)
- 3. As a set of coupled higher-order equations.

Single Mass-spring-damping example

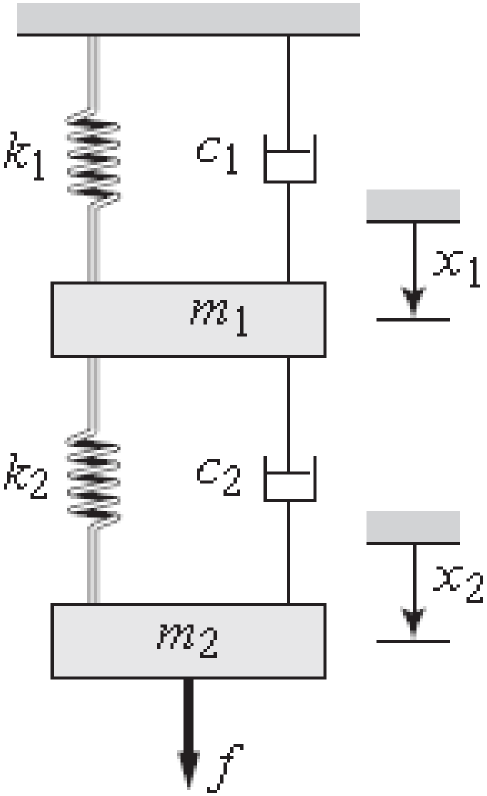
$$m\ddot{x} + c\dot{x} + kx = f$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{m}(f - kx_1 - cx_2)$$

Where is the input, system state, and output?

How to obtain a model in state-variable form from reduced form equation

Figure 5.1.1 A two-mass system.



$$5\ddot{x}_1 + 12\dot{x}_1 + 5x_1 - 8\dot{x}_2 - 4x_2 = 0$$

$$3\ddot{x}_2 + 8\dot{x}_2 + 4x_2 - 8\dot{x}_1 - 4x_1 = f(t)$$

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = \frac{1}{5}(-5z_1 - 12z_2 + 4z_3 + 8z_4)$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = \frac{1}{3}[4z_1 + 8z_2 - 4z_3 - 8z_4 + f(t)]$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B} f(t)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & -\dfrac{c}{m} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B} f(t)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -\frac{12}{5} & \frac{4}{5} & \frac{8}{5} \\ 0 & 0 & 0 & 1 \\ \frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & -\frac{8}{3} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{3} \end{bmatrix} \qquad \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$$

Output equation

- What is the system output?
- Force exerted on the single mass?

Outputs in two-mass-spring-damping system

Problem

Consider the two-mass model of Example 5.1.1.

a) Suppose the outputs are x_1 and x_2. Determine the output matrices $\mathbf{C}$ and $\mathbf{D}$. b) Suppose the outputs are $(x_2 - x_1)$, $\dot{x}_2$, and f. Determine the output matrices $\mathbf{C}$ and $\mathbf{D}$.

Standard form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

How to obtain a model in state-variable form with numerator dynamics

$$\dot{z} + 2z = 5\dot{u} + 3u$$

$$\frac{Z(s)}{U(s)} = \frac{5s+3}{s+2}$$

$$\begin{cases} \dot{x} = -2x - 7u \\ z = x + 5u \end{cases}$$

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5.2 state-variable methods with matlab

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Conversion between different forms

- The decision whether to use a reduced-form model (which is equivalent to a transfer function model) or a state-variable model depends on many factors, including personal preference.
- Different forms of model could be converted to each other easily ,especially in matlab:

tf, ss, tf2ss, ss2tf, ssdata, tfdata,

```
sys = ss(A, B, C, D)

[A, B, C, D] = ssdata(sys)

sys = tf(num,den)
```

```
sys2=tf(sys1)

sys1=ss(sys2)

[num, den] = tfdata(sys, 'v')
```

Practice matlab

- System1

$$\frac{X(s)}{F(s)} = \frac{1}{5s^2 + 7s + 4}$$

- System2

$$8\frac{d^3x}{dt^3} - 3\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 4\frac{d^2f}{dt^2} + 3\frac{df}{dt} + 5f$$

■ Problem

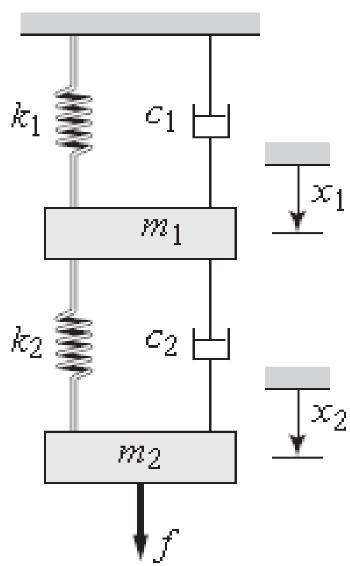
Obtain the transfer functions $X_1(s)/F(s)$ and $X_2(s)/F(s)$ of the state-variable model obtained in Example 5.1.3. The matrices and state vector of the model are

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -\frac{12}{5} & \frac{4}{5} & \frac{8}{5} \\ 0 & 0 & 0 & 1 \\ \frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & -\frac{8}{3} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{3} \end{bmatrix}$$

and

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$$

Figure 5.1.1 A two-mass system.



Linear ode solver

Table 5.2.1 LTI object functions.

Command	Description
`sys = ss(A, B, C, D)`	Creates an LTI object in state-space form, where the matrices A, B, C, and D correspond to those in the model $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, $\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$.
`[A, B, C, D] = ssdata(sys)`	Extracts the matrices A, B, C, and D of the LTI object `sys`, corresponding to those in the model $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, $\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$.
`sys = tf(num,den)`	Creates an LTI object in transfer function form, where the vector `num` is the vector of coefficients of the transfer function numerator, arranged in descending order, and `den` is the vector of coefficients of the denominator, also arranged in descending order.
`sys2=tf(sys1)`	Creates the transfer function model `sys2` from the state model `sys1`.
`sys1=ss(sys2)`	Creates the state model `sys1` from the transfer function model `sys2`.
`[num, den] = tfdata(sys, 'v')`	Extracts the coefficients of the numerator and denominator of the transfer function model `sys`. When the optional parameter `'v'` is used, if there is only one transfer function, the coefficients are returned as vectors rather than as cell arrays.

Linear ode solvers

- The Control System Toolbox provides several solvers for linear models. These solvers are categorized by the type of input function they can accept:

zero input(***the initial function***),

step input(***the step function***),

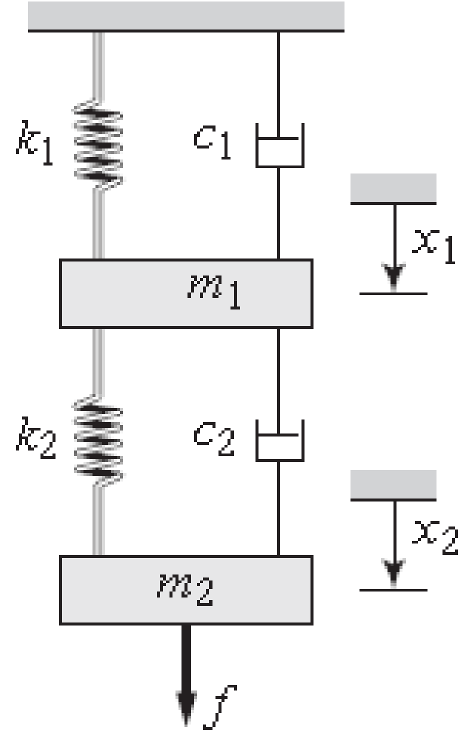
a general input function(***the lsim function***)….

- Obtaining the characteristic polynomial and the characteristic roots.

How to obtain the free response x1

- X0=[5,-3,4,2]

Figure 5.1.1 A two-mass system.



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5.3 the matlab ode function

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Introduction

- MATLAB provides functions called solvers, that implement several numerical solution methods for solving differential equations.
- The ode15s and ode45 solvers are sufficient to solve the problems encountered. It is recommended that you try ode45 first. If the equation proves difficult to solve (as indicated by a lengthy solution time or by a warning or error message), then use ode15s.

Ode solver practice

$$\dot{y} = f(t, y)$$

```
[t,y] = ode45(@ydot, tspan, y0)
```

$$5\ddot{y} + 7\dot{y} + 4y = f(t)$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{5} f(t) - \frac{4}{5} x_1 - \frac{7}{5} x_2$$

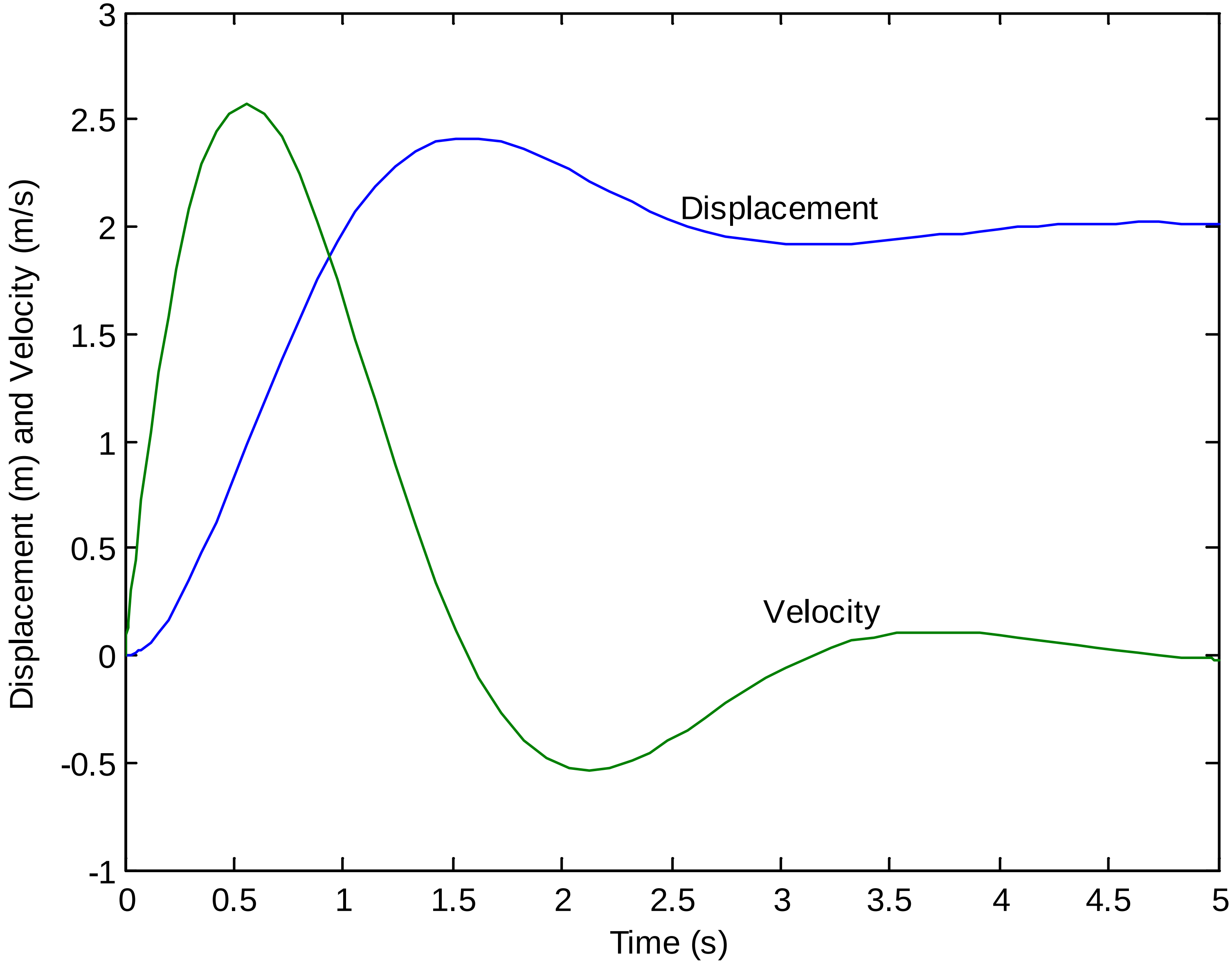
Matrix methods

- *m = 1, c = 2, k = 5,*

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{m} f(t) - \frac{k}{m} x_1 - \frac{c}{m} x_2 \qquad \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B} f(t)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



Displacement
Velocity
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time (s)
Displacement (m) and Velocity (m/s)