System Modeling and Simulation

Chapter 6. Electrical and Electromechanical Systems

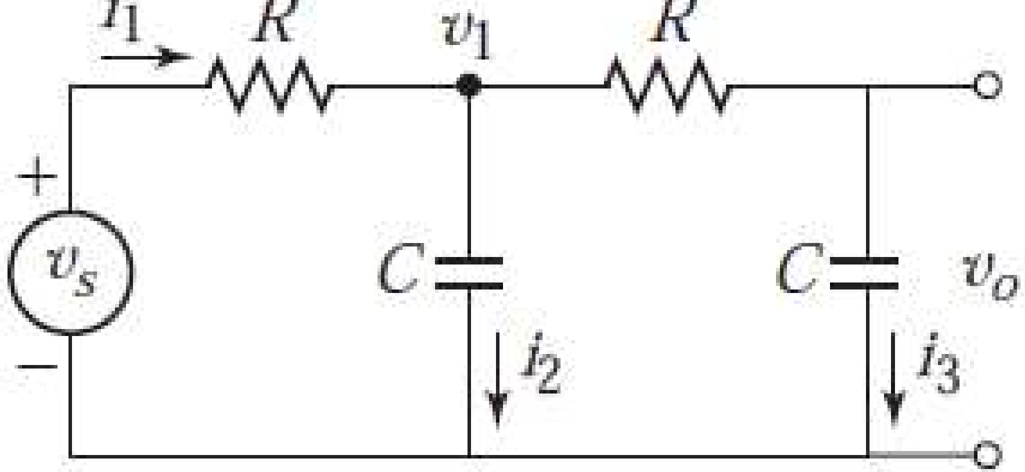
6.3 Electric Motors

邹渊
Yuan Zou
Tel: 68944115
Email: zouyuan@bit.edu.cn

Impedance

- $Z(s)$: the ratio of a voltage transform $V(s)$ to a current transform $I(s)$.
- How about resistance, capacitance and inductance?

Example



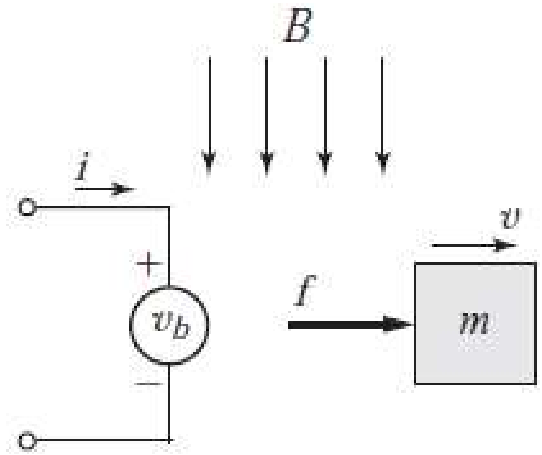
$$\frac{V_o(s)}{V_s(s)} = \frac{1}{R^2C^2s^2 + 3RCs + 1}$$

Electric motors

- Electromechanical system
- Electric + mechanical
- Motors;
- Speakers;
- Priciples for electric and mechanical system
- Magnetic field work as the media

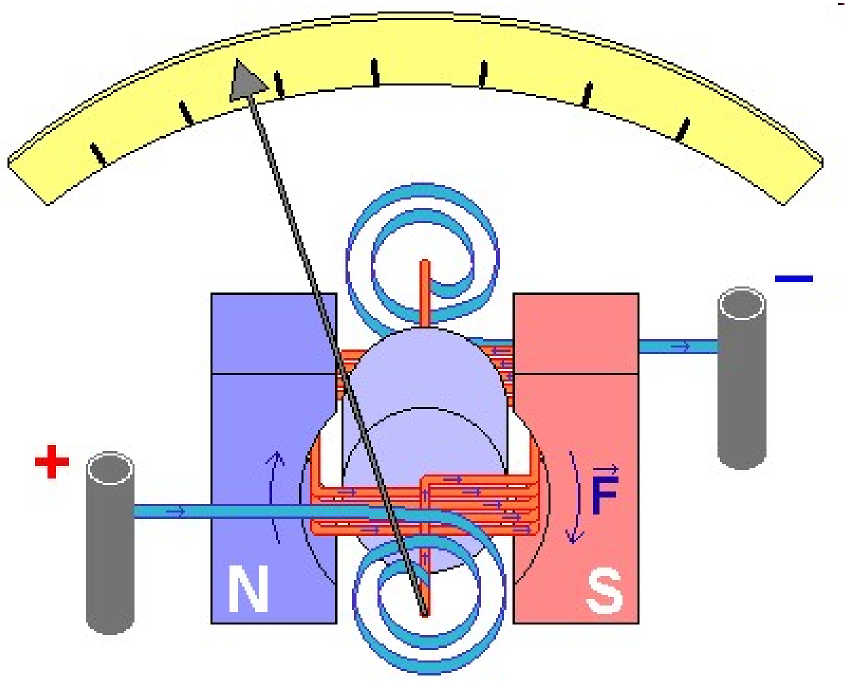
Magnetic coupling

- Basic law: 1) a force is exerted on the conductor, carrying a current in the magnetic field. 2) a voltage will be induced if the conductor moves in the field.
- f=BLi
- vb=BLv



D'Arsonval meter

- Mechanical dynamics



- Electric dynamics

- Steady state: Angle corresponds to current or voltage

Electric Motors

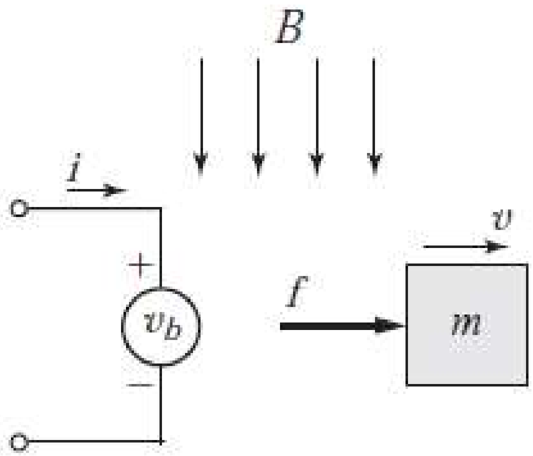
- Magnetic Coupling

Two principles:

1. $f = BLi$ the force

2. $v_b = BLv$ the voltage

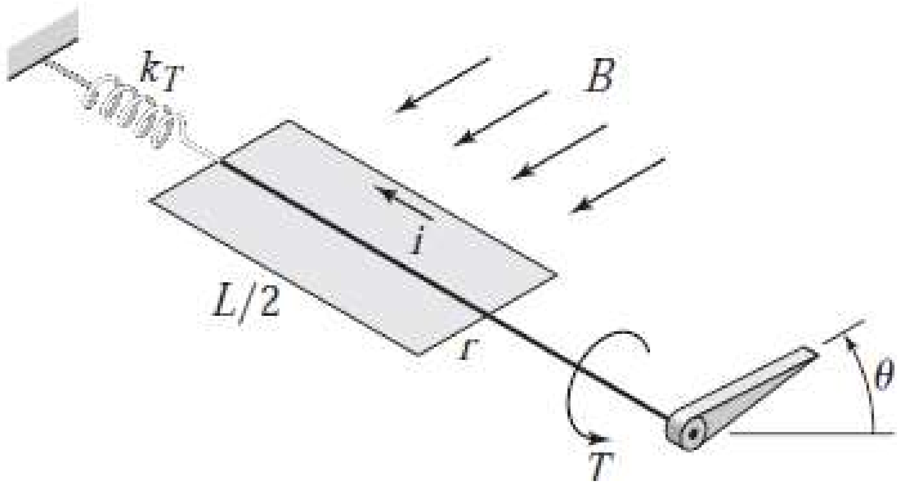
The expressions:



$$v_b i = f v = BLiv$$

$$m\dot{v} = f = BLi$$

- The D'arsonval Meter



The interaction between the current and the field produces a torque that tends to rotate the coil and pointer.

DC motors

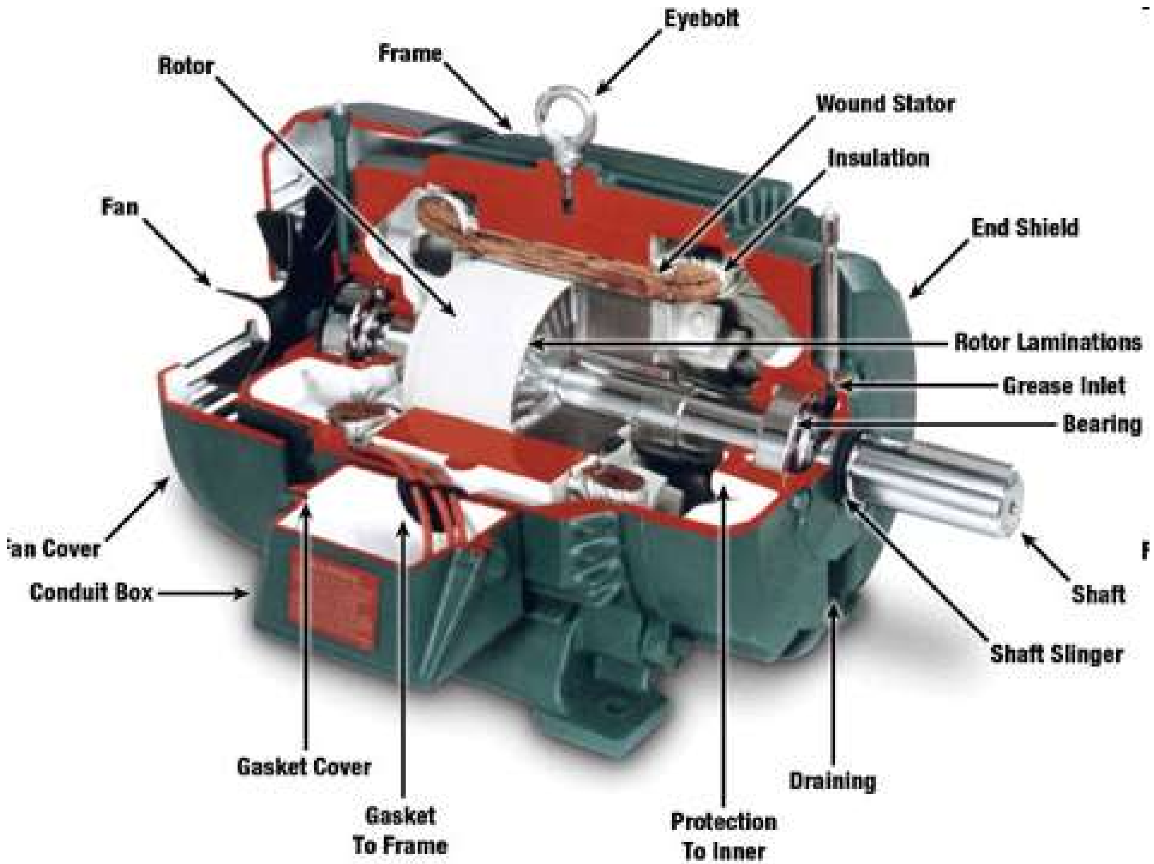


Figure 8 - Motor Construction

- Stators-> magnetic field
- Rotors-> Torque output
- Amarture-> Attached to Rotors
- Commutaters-> Connecting to amarture

Armature-controlled DC motor

- $T=(n*B*L*ia)*r=K_T*ia$ K_T-Torque constant
- $vb=n*B*L*v=(n*b*L*r)*w=K_b w$

  K_b-back emf constant
- Equations：

  $va-Ra*ia-La*d(ia)/dt-k_b w=0$

  $I*dw/dt=T-c*w-TL=Kt*ia-cw-TL$

Kt=Kb=0.05N.m/A； c=1e-3N.m.s/rad；
Ra=0.5; La=2*e-3H; I=9e-5 kg*m^2
TL=0;
Obtain step response ia(t) and w(t) when the applied voltage is va=10V

Field controlled DC motor

- T=n*B(if)*L*ia*r=(n*L*r*ia)*B(if)=T(if)
- $T=K_T*if$
- Equations

vf=Rf*if+Lf*d(if)/dt

I*dw/dt=T-c*w-TL=Kt*if-cw-TL

- Model of an armature-controlled dc motor

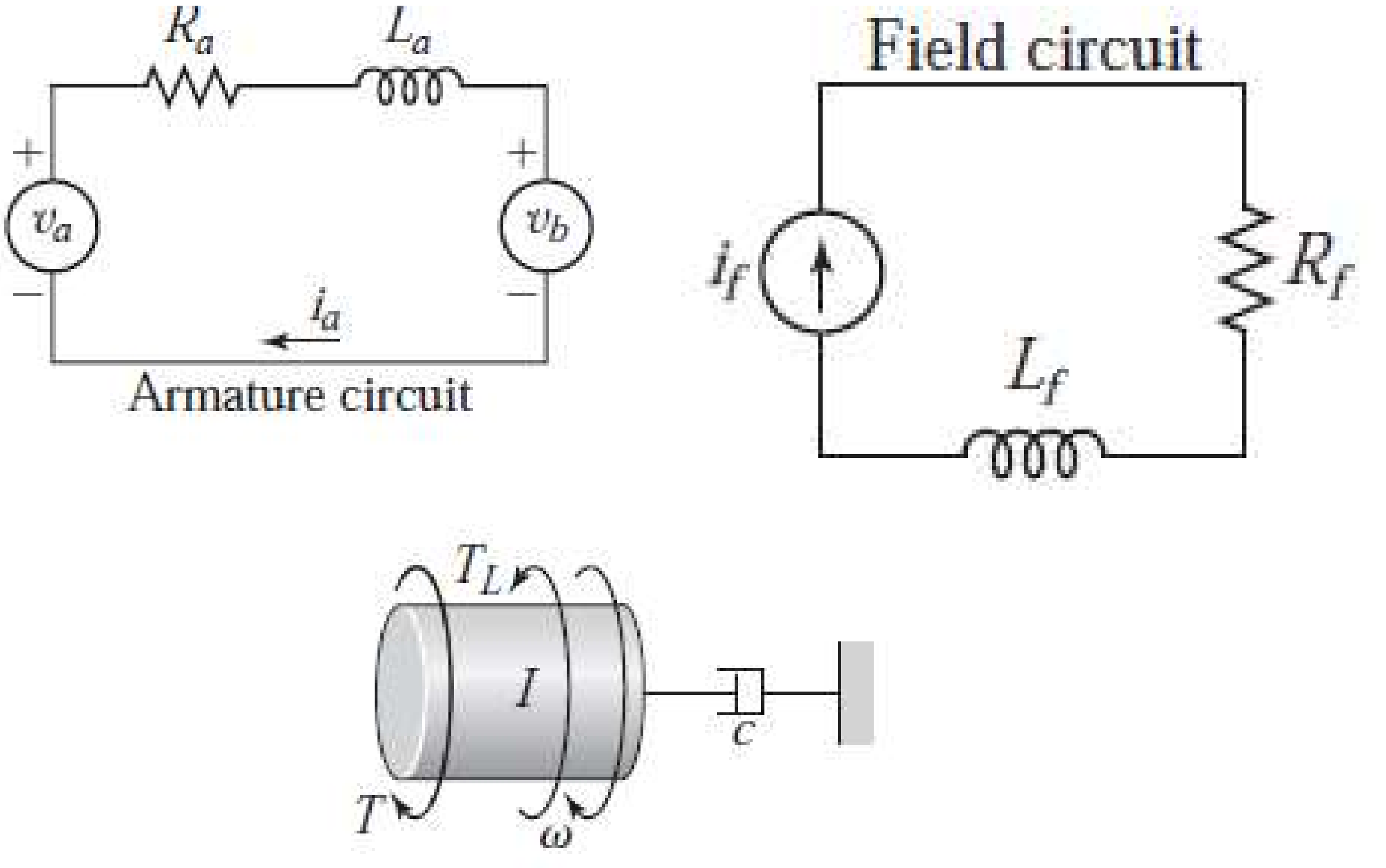


Diagram of an armature-controlled dc motor

Transfer function and state-variable from

$$\frac{I_a(s)}{V_a(s)} = \frac{Is + c}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{I_a(s)}{T_L(s)} = \frac{K_b}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{\Omega(s)}{T_L(s)} = -\frac{L_a s + R_a}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{di_a}{dt} = \frac{1}{L_a}\left(v_a - R_a i_a - K_b \omega\right)$$

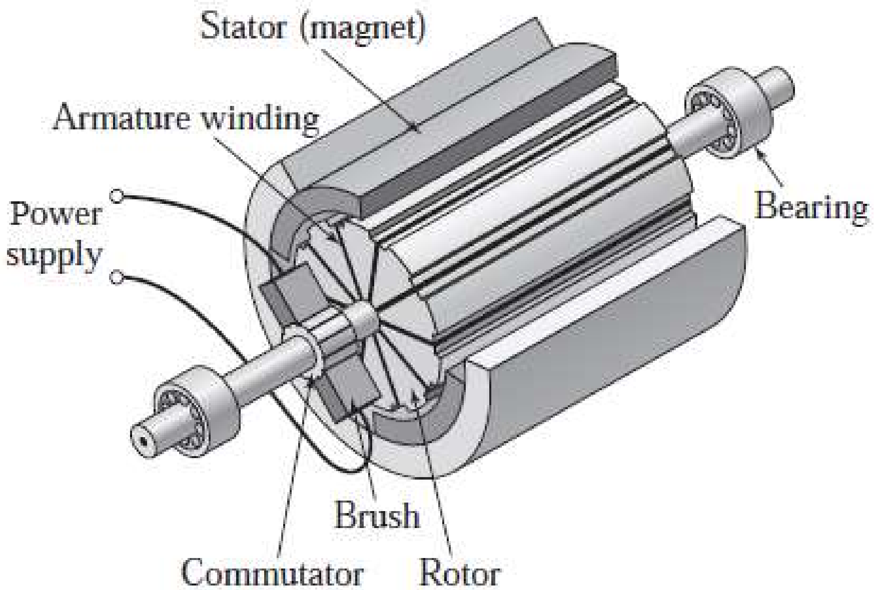
$$\frac{d\omega}{dt} = \frac{1}{I}\left(K_T i_a - c\omega - T_L\right)$$

Response of an Armature-controlled dc motors

- Kt=Kb=0.05N.m/A； c=1e-3N.m.s/rad；
  Ra=0.5; La=2*e-3H; I=9e-5 kg*m^2
  TL=0
- Obtain step response ia(t) and w(t) when the applied voltage is va=10V

- Dc Motors

There are many types of electric motors, but the two main categories are *direct current (dc)* motors and *alternating current (ac)* motors. Within the dc motor category there are the armature-controlled motor and the field controlled motor.



Cutaway view of a permanent magnet motor.

Using the principle 1, the torque on the armature is

$$T = (nBLi_a)r = (nBLr)i_a = K_T i_a$$

where *KT =nBLr* is the motor's *torque constant*.

The motion of a current-carrying conductor in a field produces a voltage in the conductor that opposes the current. This voltage in the armature is called the *back emf.*

$$v_b = nBLv = (nBLr)\omega = K_b\omega$$

$K_b = nBLr$ is the motor's *back emf constant,* Note that the expressions for *KT* and *Kb* are identical and thus, *KT* and *Kb* have the same numerical value if expressed in the same units.

For this reason, motor manufacturers usually do not give values for *Kb*.

The back emf is a voltage drop in the armature circuit. Thus, Kirchhoff's voltage law gives

$$v_a - R_a i_a - L_a \frac{di_a}{dt} - K_b \omega = 0 \qquad (1)$$

From Newton's law applied to the inertia *I* ,

$$I \frac{d\omega}{dt} = T - c\omega - T_L = K_T i_a - c\omega - T_L \qquad (2)$$

Equations (1) and (2) constitute the system model

- Motor Transfer Functions

Normally we are interested in both the motor speed ω and the current i_a. The two inputs are the applied voltage v_a and the load torque T_L .

- Thus there are four transfer functions for the motor, these transfer functions can be obtained by transforming (1) and (2).

For $I_a(s)$

$$\frac{I_a(s)}{V_a(s)} = \frac{Is + c}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{I_a(s)}{T_L(s)} = \frac{K_b}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

For the output $\Omega(s)$

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{\Omega(s)}{T_L(s)} = -\frac{L_a s + R_a}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

The denominator is the same in each of the motor's four transfer functions. It is the characteristic polynomial and it gives the characteristic equation:

$$L_a I s^2 + (R_a I + c L_a)\, s + c R_a + K_b K_T = 0$$

- State-Variable Form of the Motor Model

Equations (1) and (2) can be put into state variable form by isolating the derivatives of the state variables *ia* and *ω*.

$$\frac{di_a}{dt} = \frac{1}{L_a}(v_a - R_a i_a - K_b \omega)$$

$$\frac{d\omega}{dt} = \frac{1}{I}(K_T i_a - c\omega - T_L)$$

Note that these state variables describe the energies $L i_a^2/2$ and $I\omega^2/2$ stored in the system.

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6.4 Analysis of Motor Performance

邹渊
Yuan Zou
Tel: 68944115
Email: zouyuan@bit.edu.cn

Analysis of Motor Performance

Now using the transfer functions model of an armature-controlled dc motor to investigate the performance of such motors.

$$\frac{I_a(s)}{V_a(s)} = \frac{Is + c}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{I_a(s)}{T_L(s)} = \frac{K_b}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

$$\frac{\Omega(s)}{T_L(s)} = -\frac{L_a s + R_a}{L_a I s^2 + (R_a I + cL_a)\, s + cR_a + K_b K_T}$$

- Motor and Amplifier Performance

In evaluating the performance of a motion-control system, the following are important:

1. The energy loss per cycle is

$$E = \int_0^{t_f} R i^2(t)\, dt + \int_0^{t_f} c\omega^2(t)\, dt \quad (1)$$

2. Maximum required current and motor torque, i_{max} and T_{max}.

3. Maximum required motor speed, ω_{max}.

4. Maximum required voltage, v_{max}.

5. Average required current and motor torque, *i*rms and *T*rms.

The root mean square of torque is calculated as follows

$$T_{\text{rms}} = \sqrt{\frac{1}{t_f}\int_0^{t_f} T^2(t)\, dt} \qquad (2)$$

Note that *i*rms = *T*rms/*KT* .

- 6. Maximum speed error: This is the maximum difference between the desired speed given by the profile and the actual speed.

In the following we assume that the damping constant *c* is zero. The motor model is:

$$v = Ri + L\frac{di}{dt} + K_b\omega$$
$$I\frac{d\omega}{dt} = K_T i - T_d \qquad (3)$$

The speed ω is the *motor* speed.
The *load* speed is $\omega L = \omega/N$.
The torque $Td = TL/N$.
The $I = Im + IL/N2$.

- Energy Loss

With *c* = 0, $$E = \int_0^{t_f} R i^2(t)\, dt = R \int_0^{t_f} \left(\frac{I\dot{\omega} + T_d}{K_T} \right)^2 dt$$

where (3) has been used to substitute for *I*

Assuming that *Td* is constant,

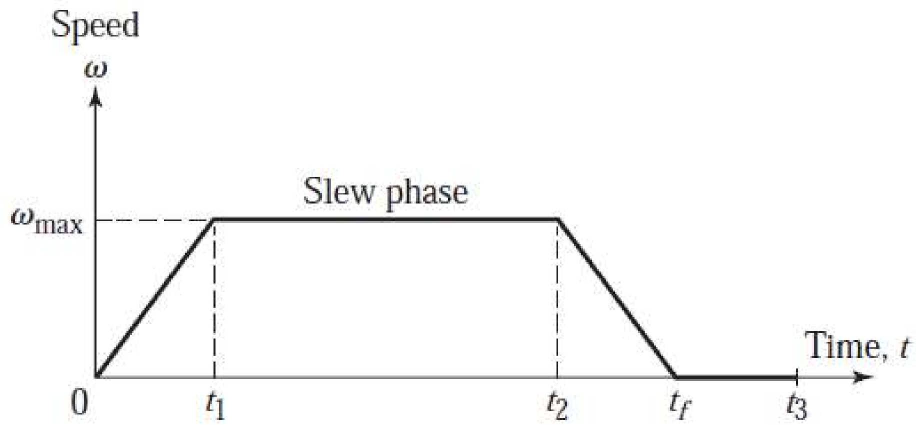
$$E = \frac{RI^2}{K_T^2} \int_0^{t_f} \dot{\omega}^2\, dt + \frac{2RIT_d}{K_T^2} \int_0^{t_f} \dot{\omega}\, dt + \frac{RT_d^2}{K_T^2} \int_0^{t_f} dt$$

- With the assumption *ω(*0*)* = *ω(t f)*, the second integral is zero. Thus

$$E = \frac{RI^2}{K_T^2} \int_0^{t_f} \dot{\omega}^2\, dt + \frac{RT_d^2 t_f}{K_T^2}$$

- # Maximum Motor Speed

If the load speed ωL *(t)* is specified instead, we can find $\omega(t)$ from $N\omega L$ *(t)*. Assuming that $\theta(0) = 0$, and because $t_f - t_2 = t_1$,



$$\theta(t_f) = \theta_f = \int_0^{t_f} \omega\, dt = 2\left(\frac{1}{2}\omega_{\max} t_1\right) + \omega_{\max}(t_2 - t_1) = \omega_{\max} t_2$$

Thus

$$\omega_{\max} = \frac{\theta_f}{t_2}$$

- Maximum Motor Torque

The maximum required acceleration is

$$\alpha_{\max} = \frac{\omega_{\max}}{t_1} = \frac{\theta_f}{t_1 t_2}$$

Using (3) and the fact that the motor torque is *T* = *KT i*,

$$T = K_T i = I\frac{d\omega}{dt} + T_d = I\alpha + T_d$$

Thus the maximum required motor torque is

$$T_{\max} = I\alpha_{\max} + T_d = I\frac{\omega_{\max}}{t_1} + T_d = I\frac{\theta_f}{t_1 t_2} + T_d$$

- RMS Motor Torque

The rms torque is calculated by combining the equations :

$$T_{\text{rms}} = \sqrt{\frac{1}{t_f} \int_0^{t_f} T^2(t)\, dt}$$

$$T = K_T i = I \frac{d\omega}{dt} + T_d = I\alpha + T_d$$

So

$$T_{\mathrm{rms}}^2 = \frac{1}{t_f}\int_0^{t_f} T^2(t)\,dt = \frac{1}{t_f}\int_0^{t_f} (I\alpha + T_d)^2\,dt$$

This reduces to

$$T_{\mathrm{rms}}^2 = \frac{1}{t_f}\left[2I^2\alpha_{\max}^2 t_1 + T_d^2(t_1 + t_2)\right]$$

since αmax $= \theta f / t1t2$ and $t1 + t2 = t f$,

$$T_{\mathrm{rms}} = \sqrt{\frac{2I^2\theta_f^2}{t_f t_1 t_2^2} + T_d^2}$$

These equations are summarized in Table as follow.

Profile times t_1, t_2, t_f	See Figure 6.5.2
Motor displacement	θ_f = area under speed profile
Load torque felt at motor	T_d
Motor requirements	
Energy consumption/cycle	$E = \frac{R}{K_T^2}\left(\frac{2I^2\theta_f^2}{t_1 t_2^2} + T_d^2 t_f\right)$
Maximum speed	$\omega_{max} = \frac{\theta_f}{t_2}$
Maximum torque	$T_{max} = I\frac{\theta_f}{t_1 t_2} + T_d$
rms torque	$T_{rms} = \sqrt{\frac{2I^2\theta_f^2}{t_f t_1 t_2^2} + T_d^2}$
Amplifier requirements	
Maximum current	$i_{max} = \frac{T_{max}}{K_T}$
rms current	$i_{rms} = \frac{T_{rms}}{K_T}$
Maximum voltage	$v_{max} = Ri_{max} + K_b\omega_{max}$

To determine whether a given motor will be satisfactory, the calculated values of ω_{max}, T_{max}, and T_{rms} are compared with the motor manufacturer's data on maximum speed, peak torque, and rated continuous torque.