System modeling and simulation(ME340)

Chapter 8. System Analysis in the Frequency domain

8.3 Frequency response examples

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The neutrally stable case

- ??? Consider Mass-spring-damping system when damping c=0
- mx''+kx=Fsin(ωt)

$$x_{\text{free}}(t) = \frac{\dot{x}(0)}{\omega_n} \sin \omega_n t + x(0) \cos \omega_n t$$

$$x_{\text{forced}}(t) = \frac{F\omega}{m(\omega^2 - \omega_n^2)} \left(\sin \omega_n t - \frac{\omega_n}{\omega} \sin \omega t \right)$$

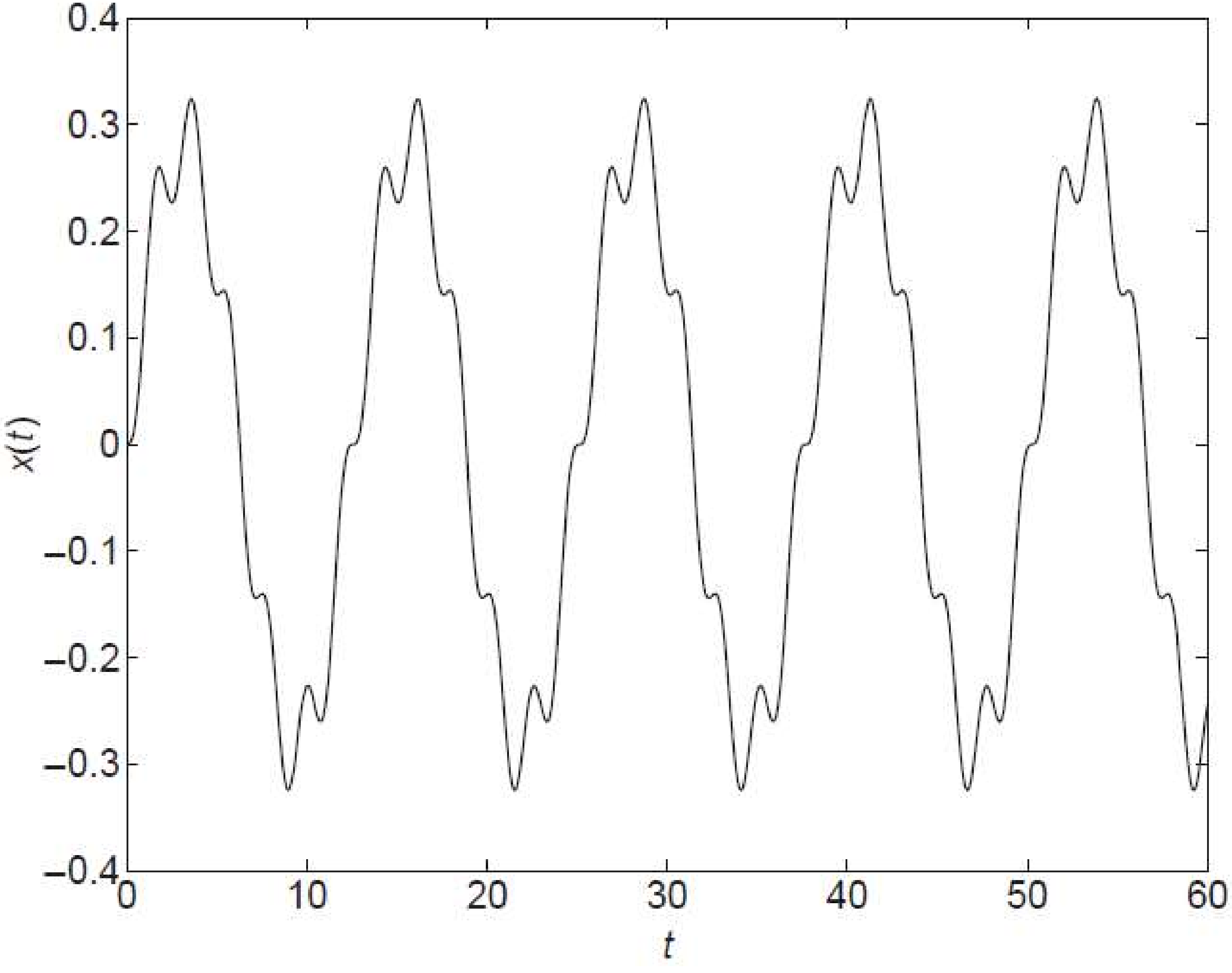
$$x(t) = \left[\frac{\dot{x}(0)}{\omega_n} - \frac{F}{k} \frac{r}{1 - r^2} \right] \sin \omega_n t + x(0) \cos \omega_n t + \frac{F}{k} \frac{1}{1 - r^2} \sin \omega t$$

Beating

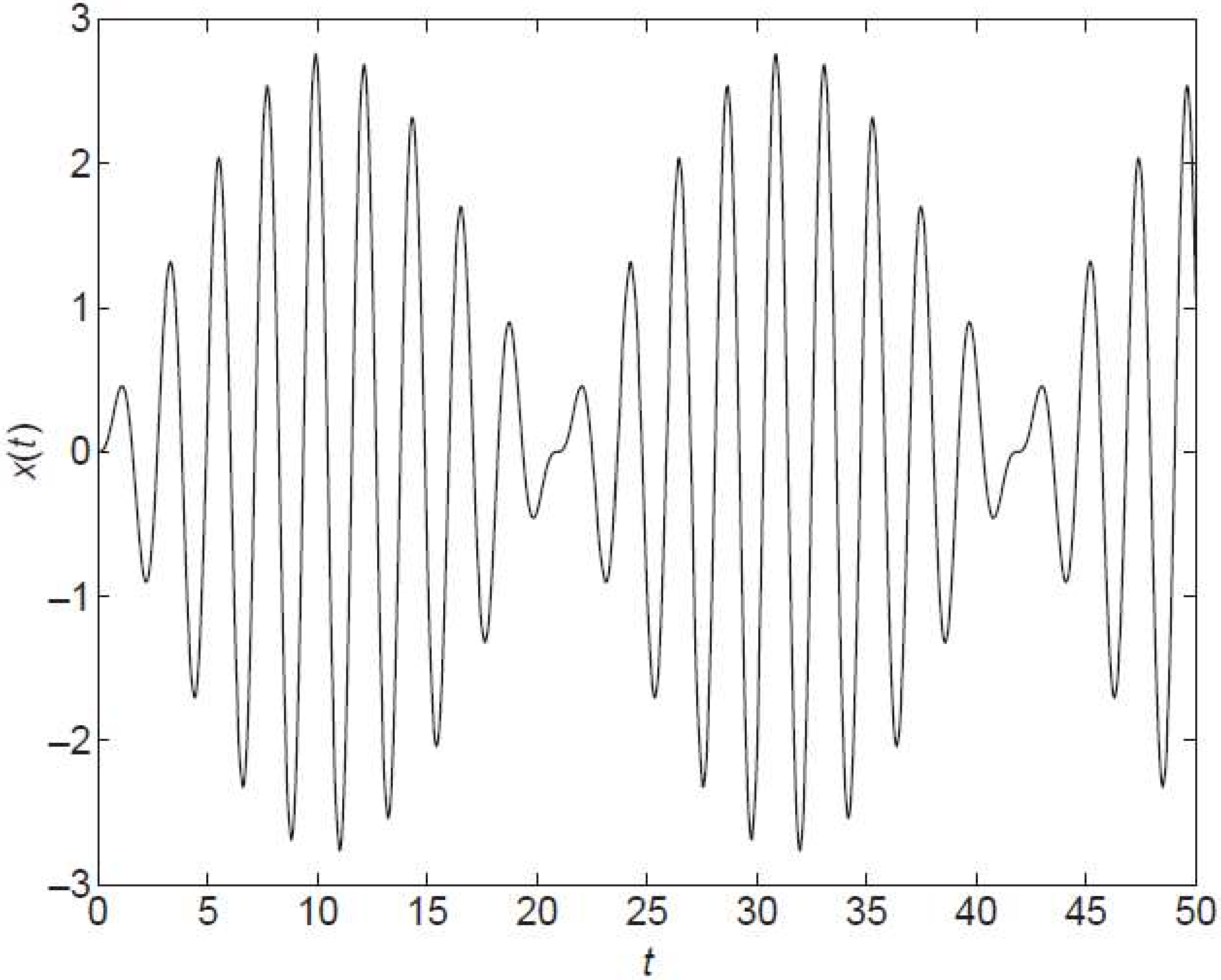
- Just consider the forced response:

$$x(t) = \frac{F}{k}\frac{1}{1-r^2}(\sin \omega t - r \sin \omega_n t)$$

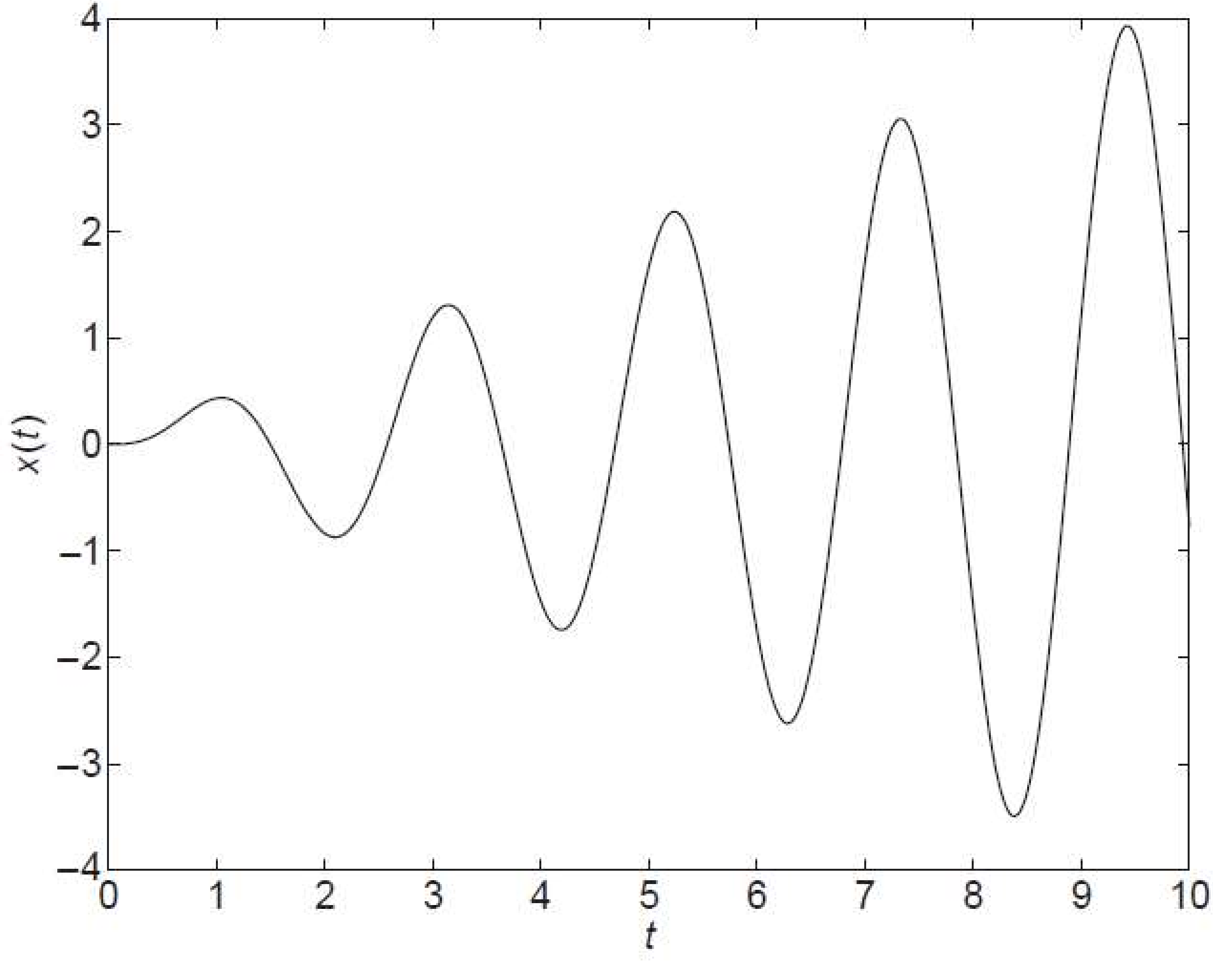
Explain this force response.



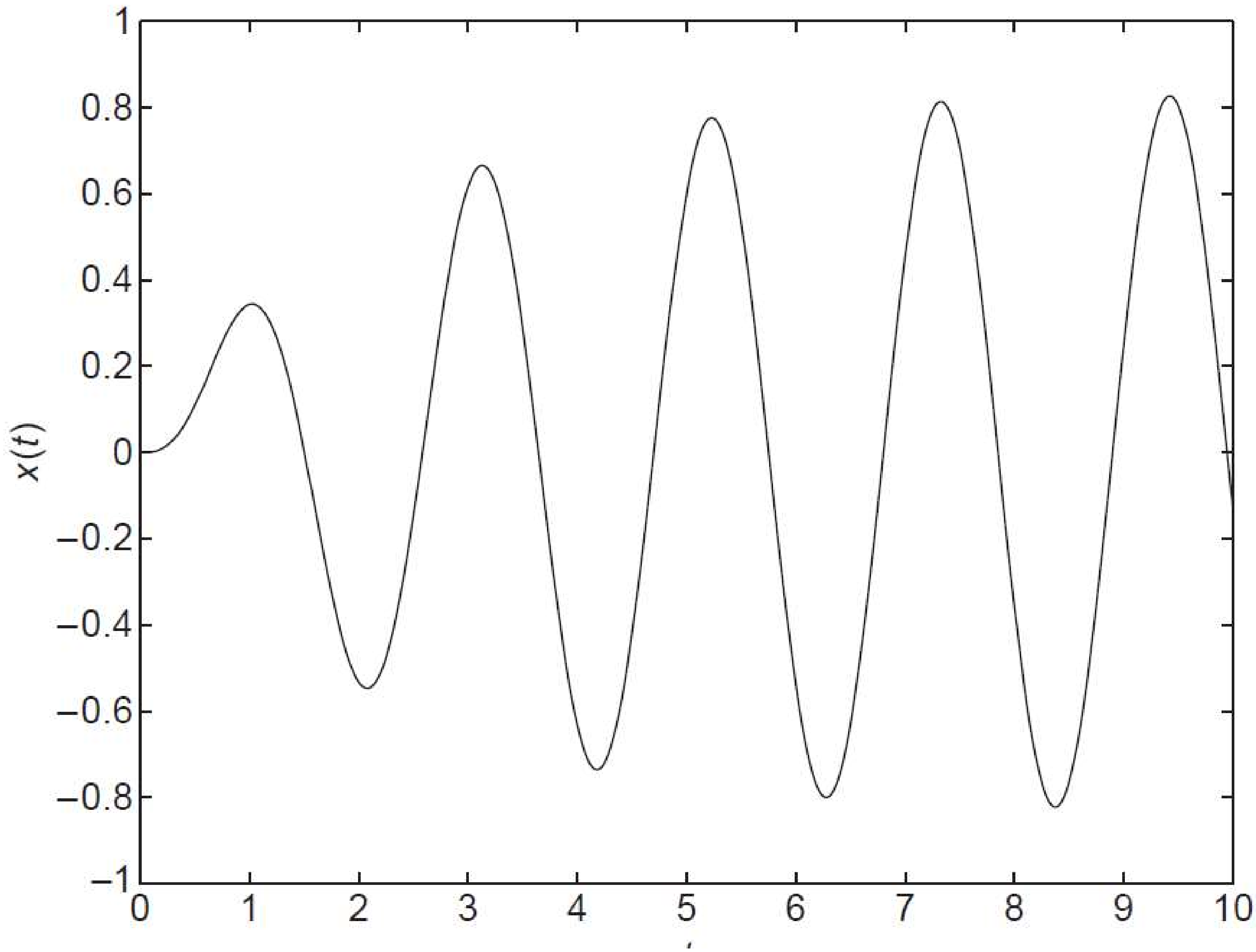
And this



What happed if r is 1?

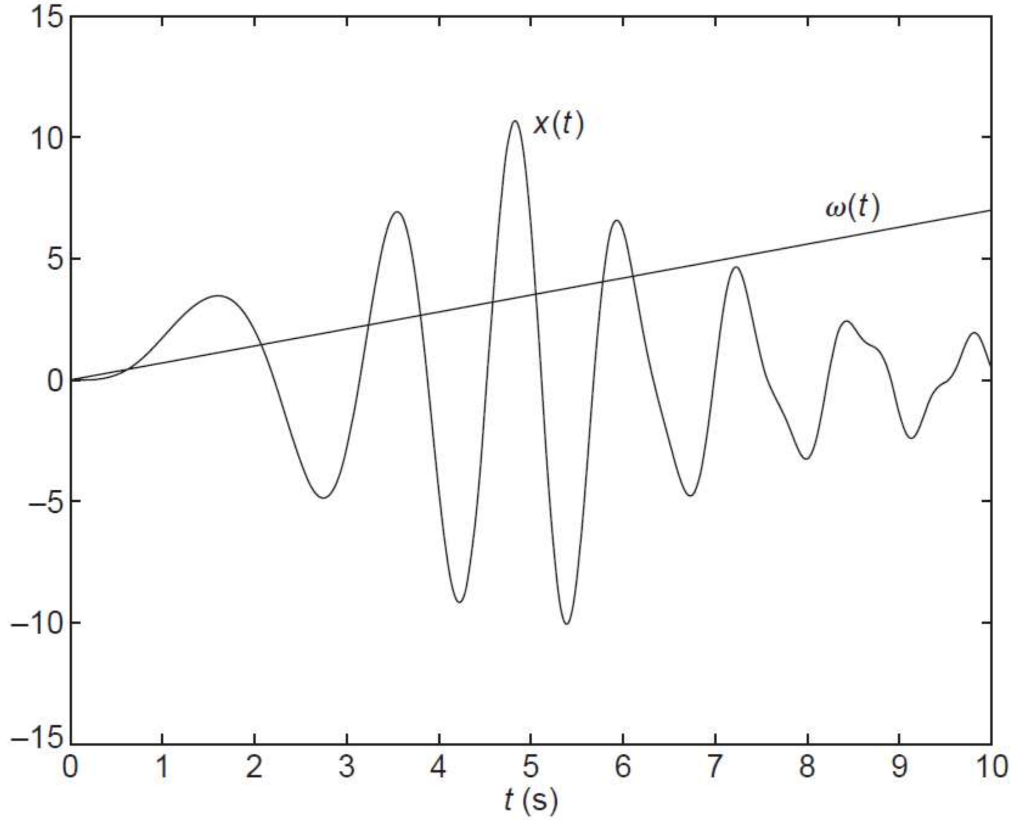


And this?

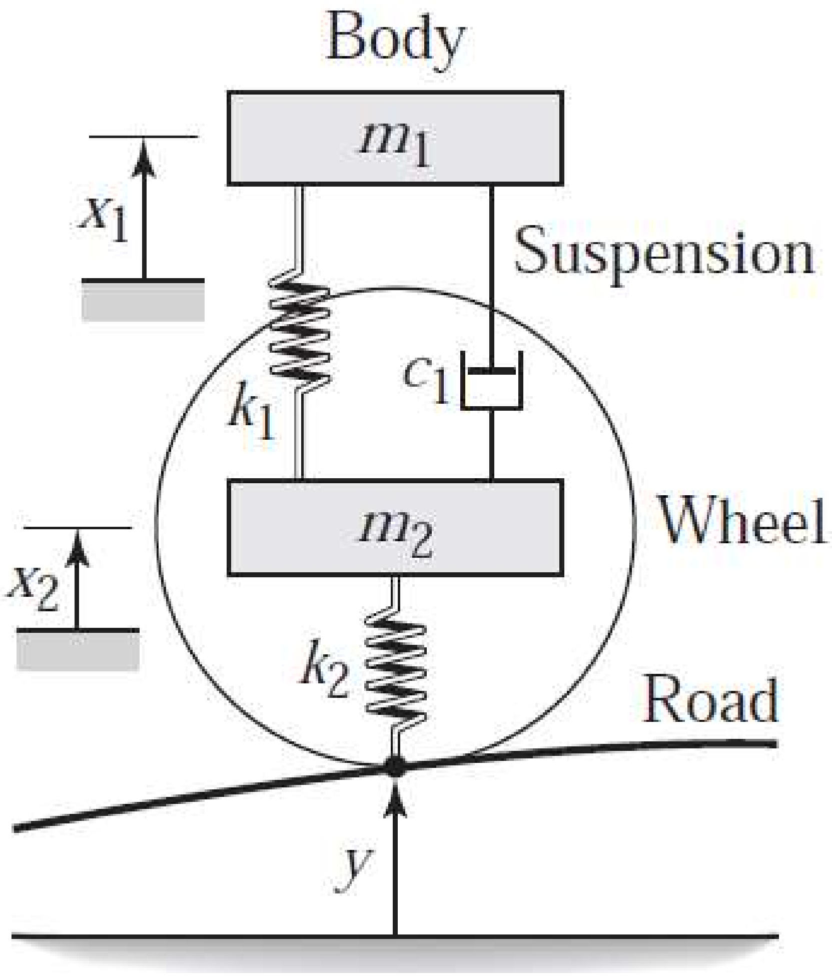


Resonance and transient response

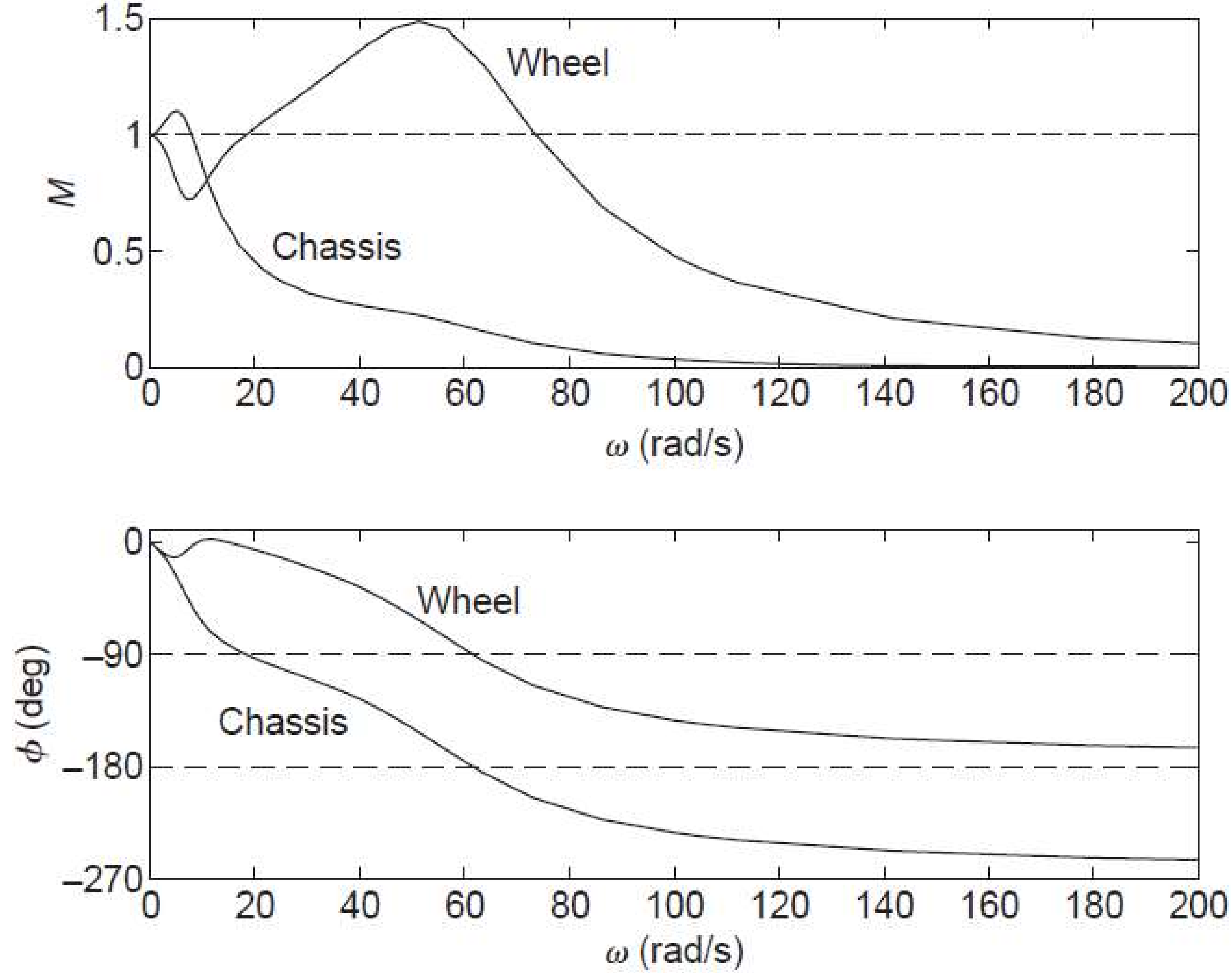
- $4x'' + 3x' + 100x = 295 * \sin(\omega(t) * t)$.



Two-mass suspension model



Bode plot


1.5
1
0.5
0
M
Wheel
Chassis
0
20
40
60
80
100
120
140
160
180
200
ω (rad/s)
0
−90
−180
−270
ϕ (deg)
Wheel
Chassis
0
20
40
60
80
100
120
140
160
180
200
ω (rad/s)

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Chapter 6. System Analysis in the Frequency domain

6.4 Filter property of dynamic system

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Filtering properties of dynamic system

- Dynamic systems response in a different way to the input with the different frequency.
- Low-pass:

  Vehicle suspension?
- High-pass:

  RC circuit?

Bandwidth

- The range of frequencies over which the power transmitted or dissipated by the system is no less than one-half of the peak power.

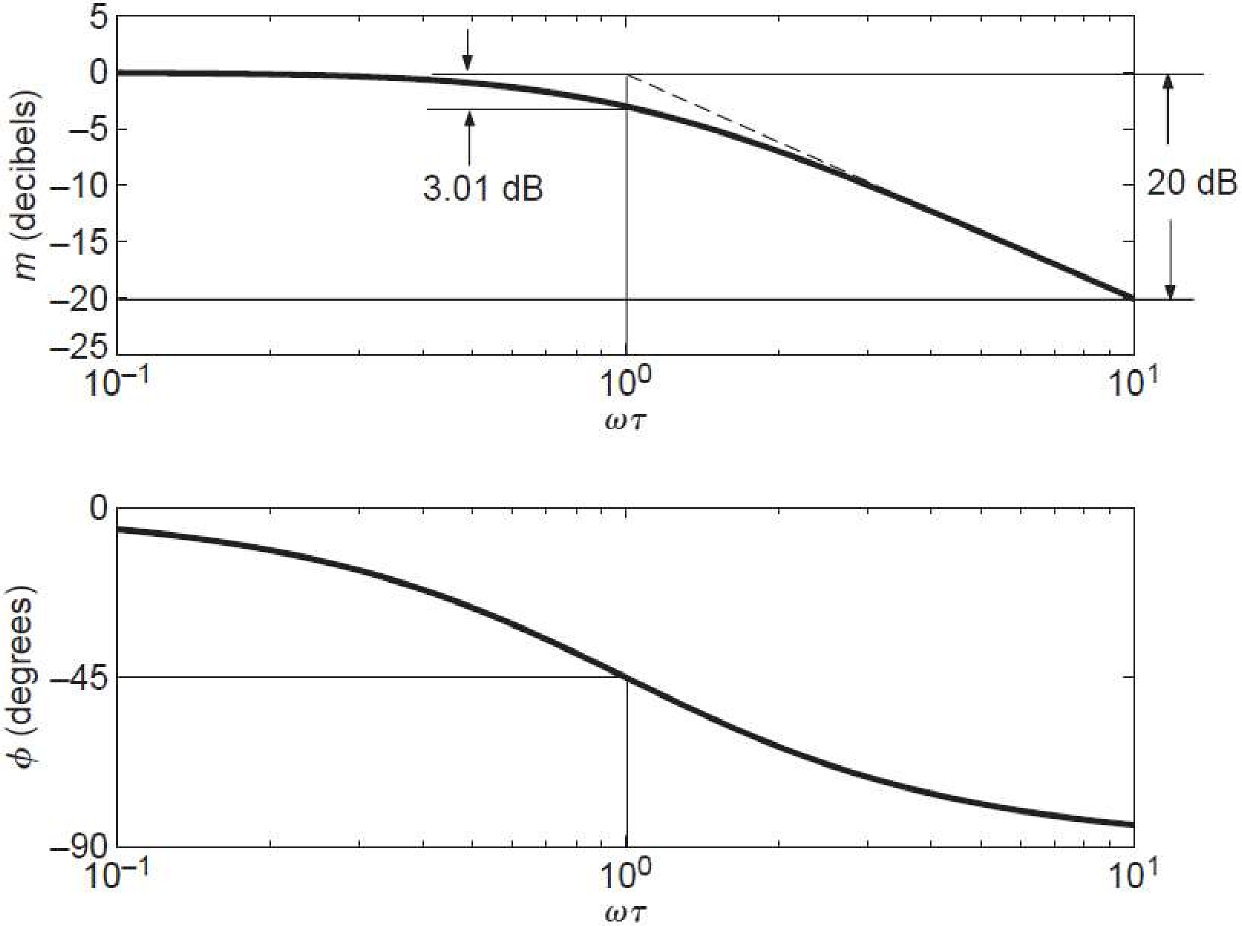
- [w1,w2]

$$M^2(\omega_1) \le \frac{M_{\text{peak}}^2}{2} \ge M^2(\omega_2)$$

Bandwidth of first-order system

- $\tau \dot{v} + v = f(t)$
- $M(\omega) = \frac{1}{\sqrt{1+(\tau\omega)^2}}$
- Band width: [0, $1/\tau$];

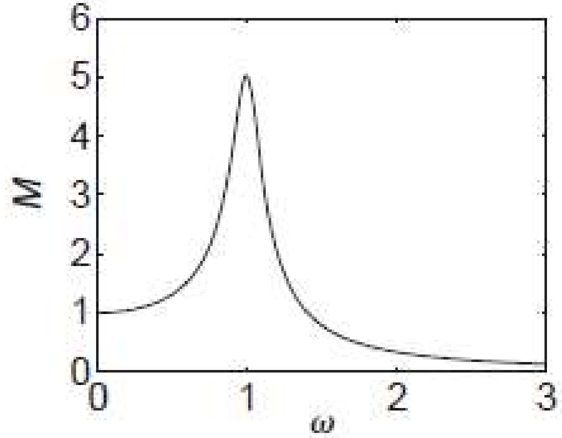
Figure 8.1.6 Asymptotes and corner frequency $\omega = 1/\tau$ of the model $1/(\tau s + 1)$.



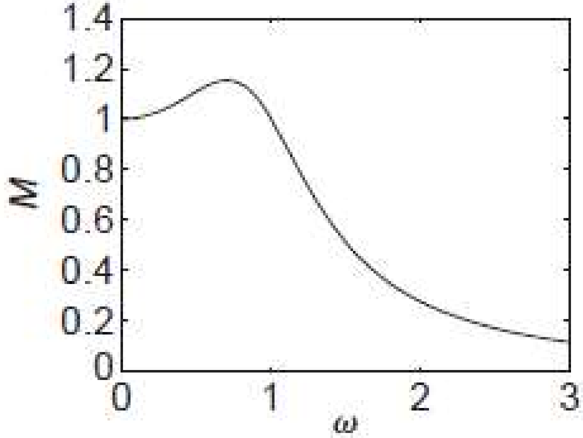
Band width of second-order system

- $m\ddot{x} + c\dot{x} + kx = f(t)$
- $M = \frac{1}{k}\frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

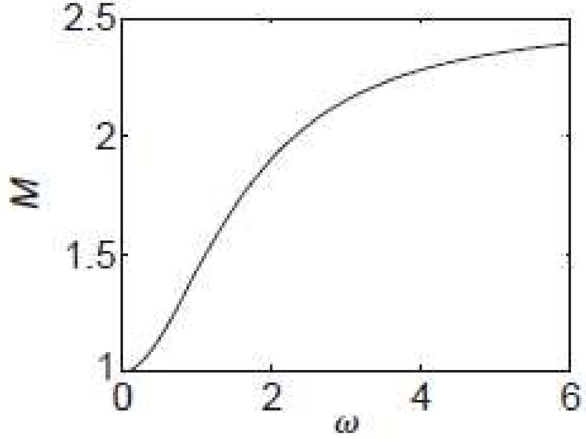
Different dynamic behavior



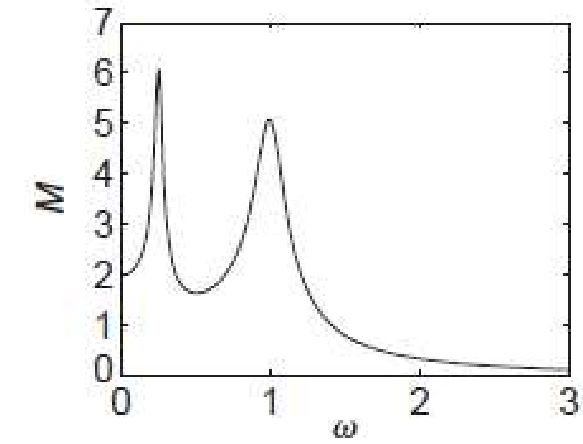
(a)



(b)



(c)



(d)

Alternative definition of bandwidth

- Often the low-frequency input might account for more power than those of high-frequency input.
- Sometimes bandwidth is defined as $[0, \omega_2]$.
- But It is not true for high-pass filter system.

Discussion for Human-ears property