System modeling and simulation(ME340)

Chapter 9. Transient response and block diagram model

9.3 Description and specification of step response

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Express the step response

- Free response

$$x(t) = Be^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

- Forced response

$$x(t) = \frac{1}{k}\left[\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t + \phi) + 1\right]$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) + \pi$$

20	$e^{-at}\sin\omega t$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at}\cos\omega t$	$\dfrac{s+a}{(s+a)^2+\omega^2}$
22	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t$	$\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
23	$-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\phi)$ $\phi=\tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{\zeta}$	$\dfrac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
24	$1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\phi)$ $\phi=\tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{\zeta}$	$\dfrac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
25	$1-\cos\omega t$	$\dfrac{\omega^2}{s(s^2+\omega^2)}$
26	$\omega t-\sin\omega t$	$\dfrac{\omega^3}{s^2(s^2+\omega^2)}$
27	$\sin\omega t-\omega t\cos\omega t$	$\dfrac{2\omega^3}{(s^2+\omega^2)^2}$

Model: $m\ddot{x} + c\dot{x} + kx = u_s(t)$

Initial conditions: $x(0) = \dot{x}(0) = 0$

Characteristic roots: $s = \dfrac{-c \pm \sqrt{c^2 - 4mk}}{2m} = -r_1, -r_2$

1. Overdamped case ($\zeta > 1$): distinct, real roots: $r_1 \neq r_2$

$$x(t) = A_1 e^{-r_1 t} + A_2 e^{-r_2 t} + \frac{1}{k} = \frac{1}{k}\left(\frac{r_2}{r_1 - r_2} e^{-r_1 t} - \frac{r_1}{r_1 - r_2} e^{-r_2 t} + 1\right)$$

2. Critically damped case ($\zeta = 1$): repeated, real roots: $r_1 = r_2$

$$x(t) = (A_1 + A_2 t)e^{-r_1 t} + \frac{1}{k} = \frac{1}{k}[(-r_1 t - 1)e^{-r_1 t} + 1]$$

3. Underdamped case ($0 \le \zeta < 1$): complex roots: $s = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$

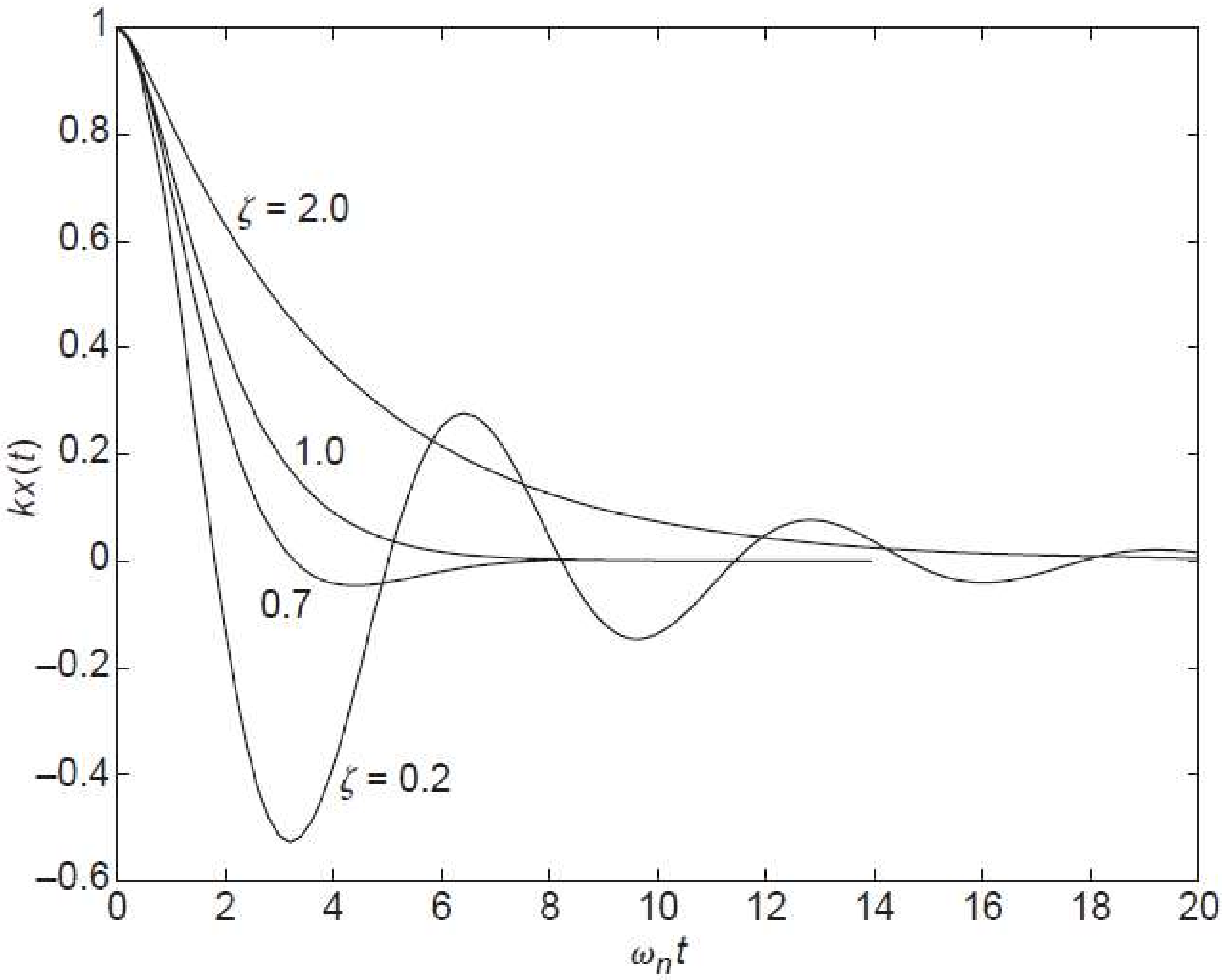
$$x(t) = Be^{-t/\tau} \sin\left(\omega_n\sqrt{1 - \zeta^2}\,t + \phi\right) + \frac{1}{k}$$

$$= \frac{1}{k}\left[\frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1 - \zeta^2}\,t + \phi\right) + 1\right]$$

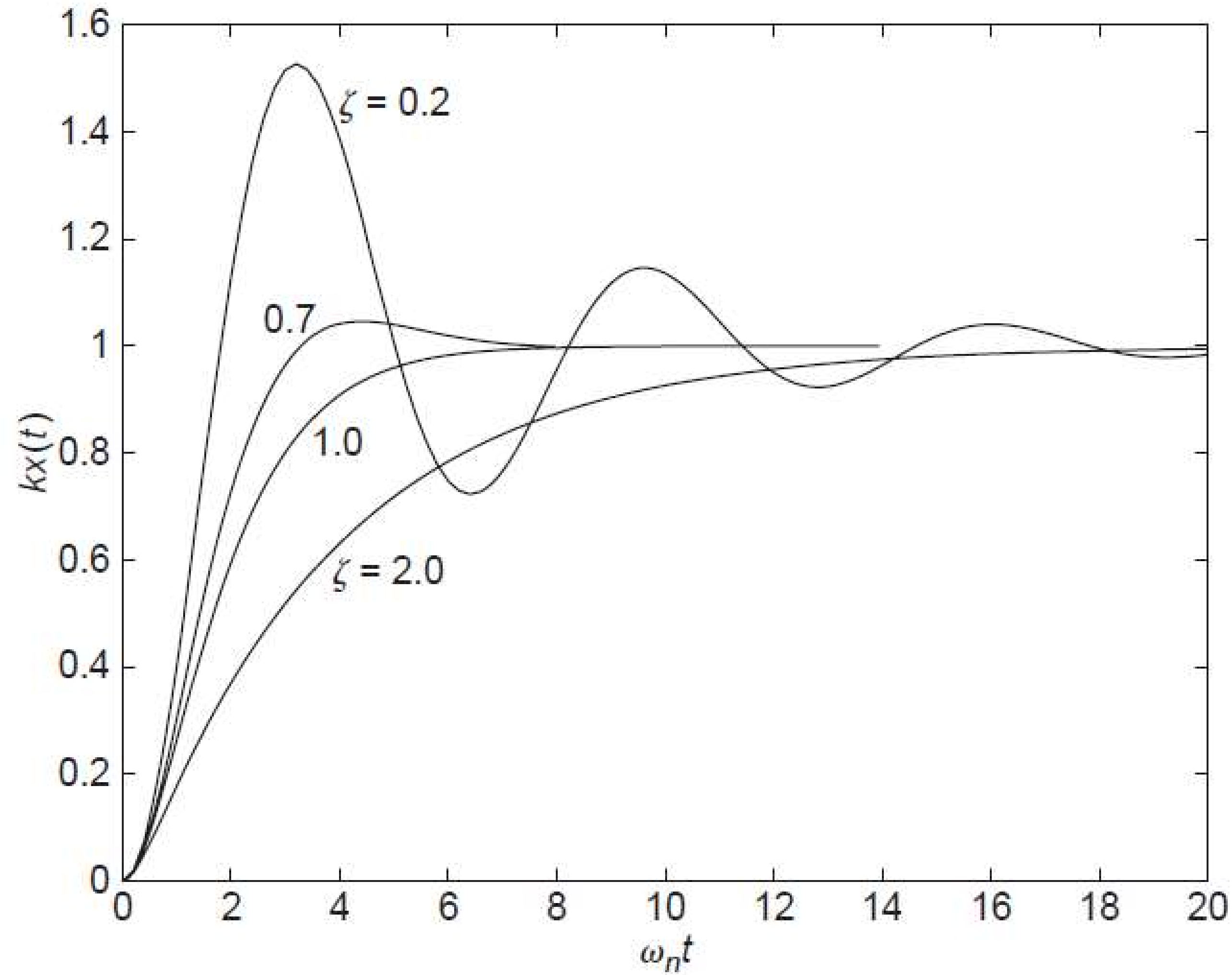
$$\phi = \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right) + \pi \qquad \text{(third quadrant)}$$

Time constant: $\tau = 1/\zeta\omega_n$

Free response： x’(0)=0

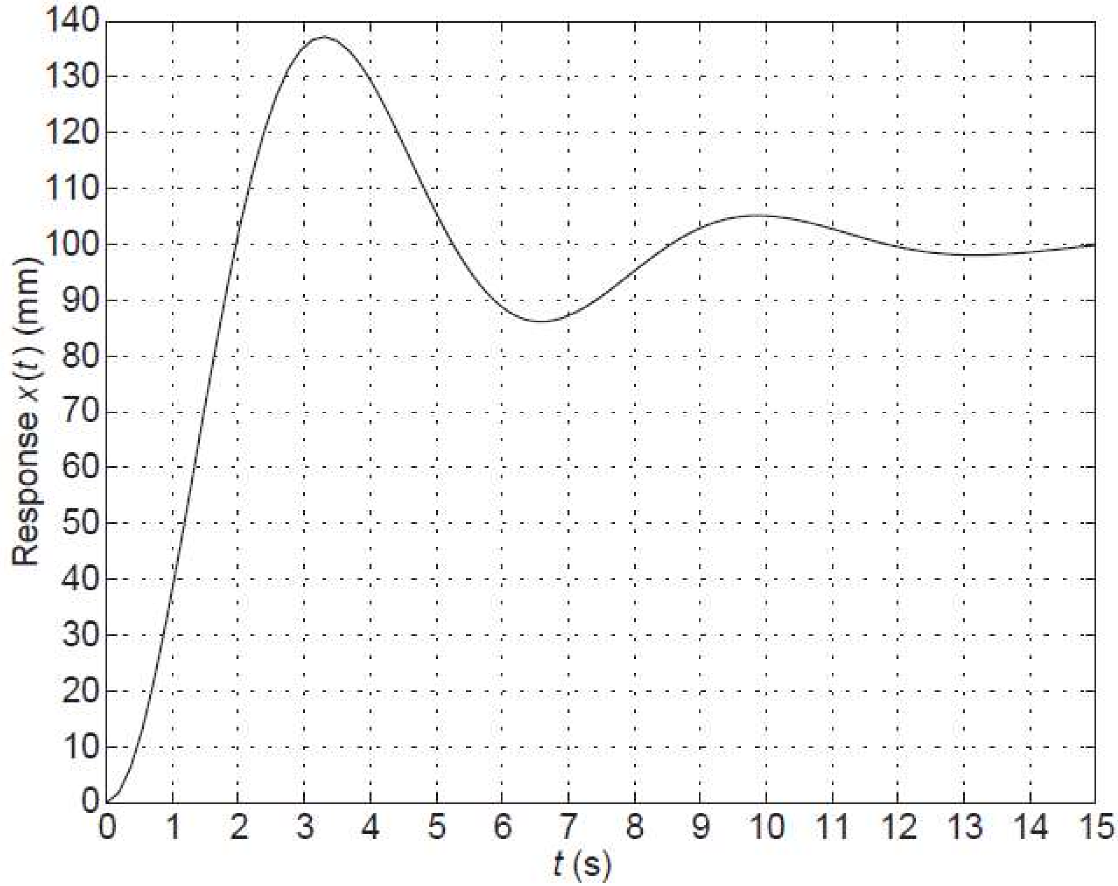


Step response： x(0)=x’(0)=0



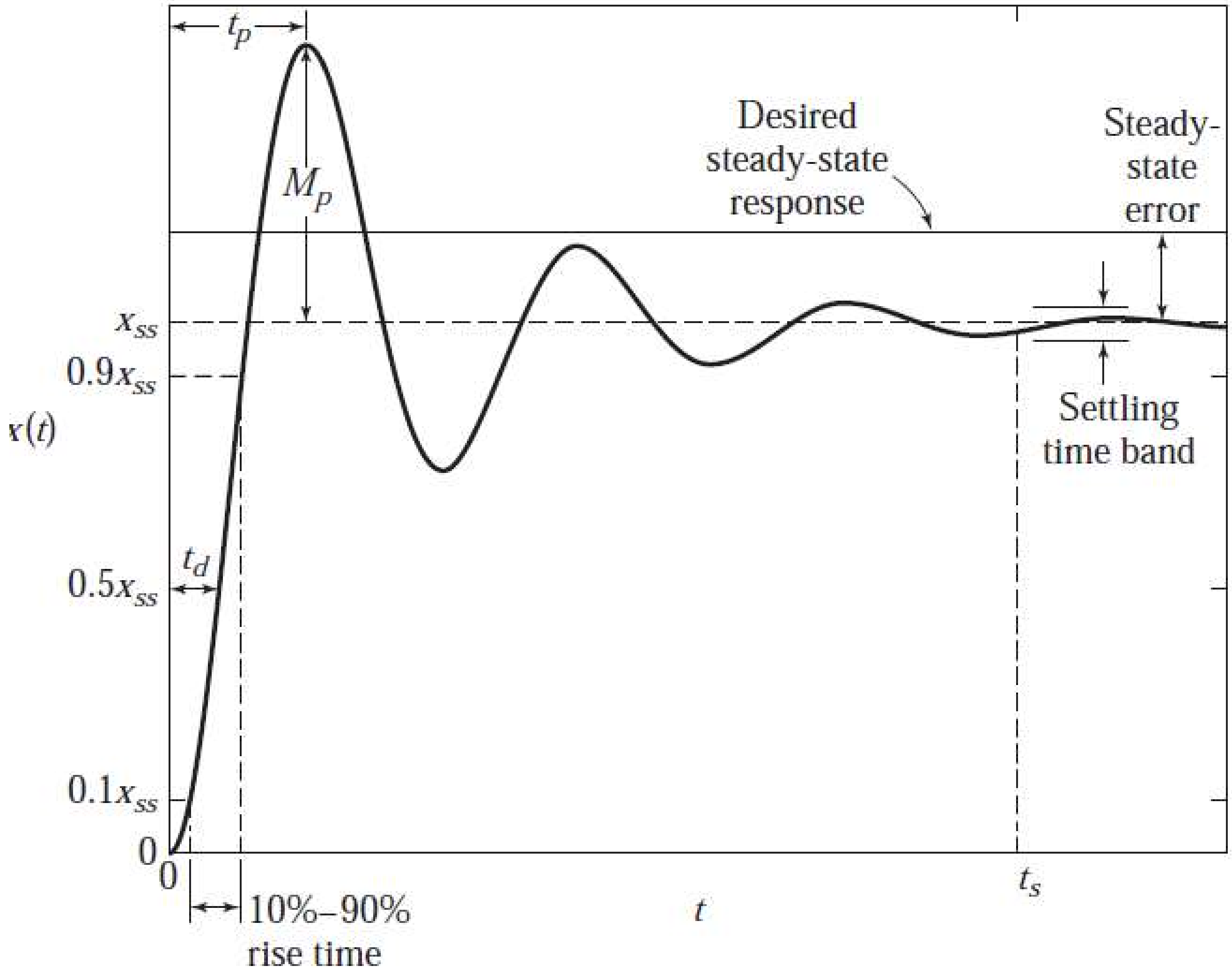
Description of step response

- How to describe the step response as the following?



Description

- Mp
- Tp
- ts
- tr
- td



Maximum overshoot

$$\frac{dx}{dt} = \frac{1}{k}\left(\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t\right) = 0$$

$$t = \frac{n\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$M_p = x_{\max} - x_{ss} = \frac{1}{k} e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$M_{\%} = \frac{x_{\max} - x_{ss}}{x_{ss}} 100 = 100 e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

Rising time

- Rising time: 100%

$$\omega_n\sqrt{1-\zeta^2}\,t+\phi=n\pi \quad n=0,1,2,\ldots$$

$$t_r=\frac{2\pi-\phi}{\omega_n\sqrt{1-\zeta^2}}$$

Setting time

$$t_s = \frac{4}{\zeta \omega_n}$$

Delay time

Set $x = 0.5x_{ss} = 0.5/k$ i

$$e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}t+\phi\right) = -0.5\sqrt{1-\zeta^2}$$

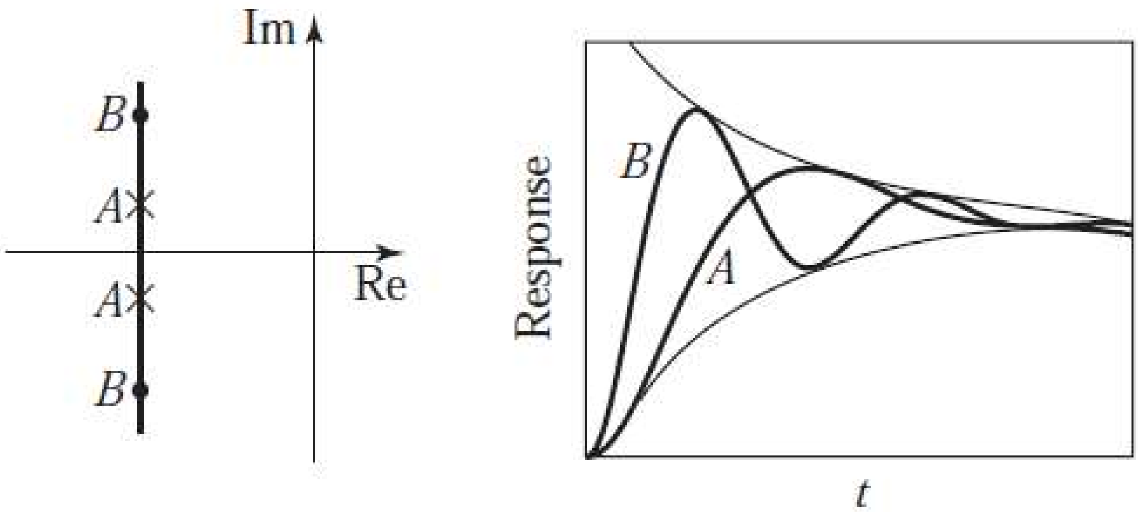
$$t_d \approx \frac{1+0.7\zeta}{\omega_n}$$

Summary

Maximum percent overshoot	$M_{\%} = 100e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
	$\zeta = \dfrac{R}{\sqrt{\pi^2 + R^2}}, \quad R = \ln \dfrac{100}{M_{\%}}$
Peak time	$t_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
Delay time	$t_d \approx \dfrac{1 + 0.7\zeta}{\omega_n}$
100% rise time	$t_r = \dfrac{2\pi - \phi}{\omega_n\sqrt{1-\zeta^2}}$
	$\phi = \tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right) + \pi$

Root location and response

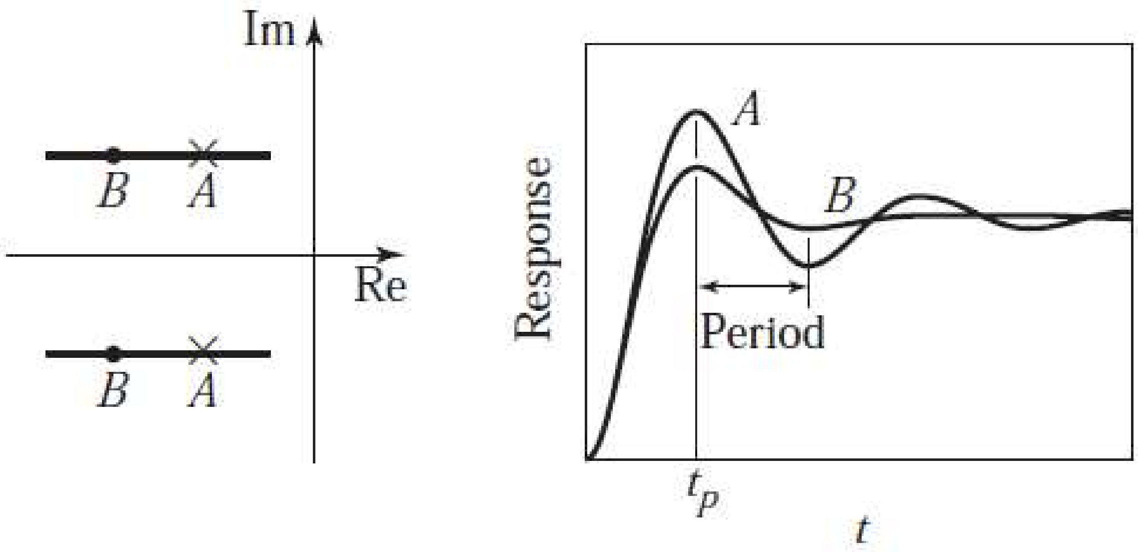
Models A and B have the same real part, the same time constant, and the same decay time.



$$x(t) = \frac{1}{k}\left[\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \phi\right) + 1\right]$$

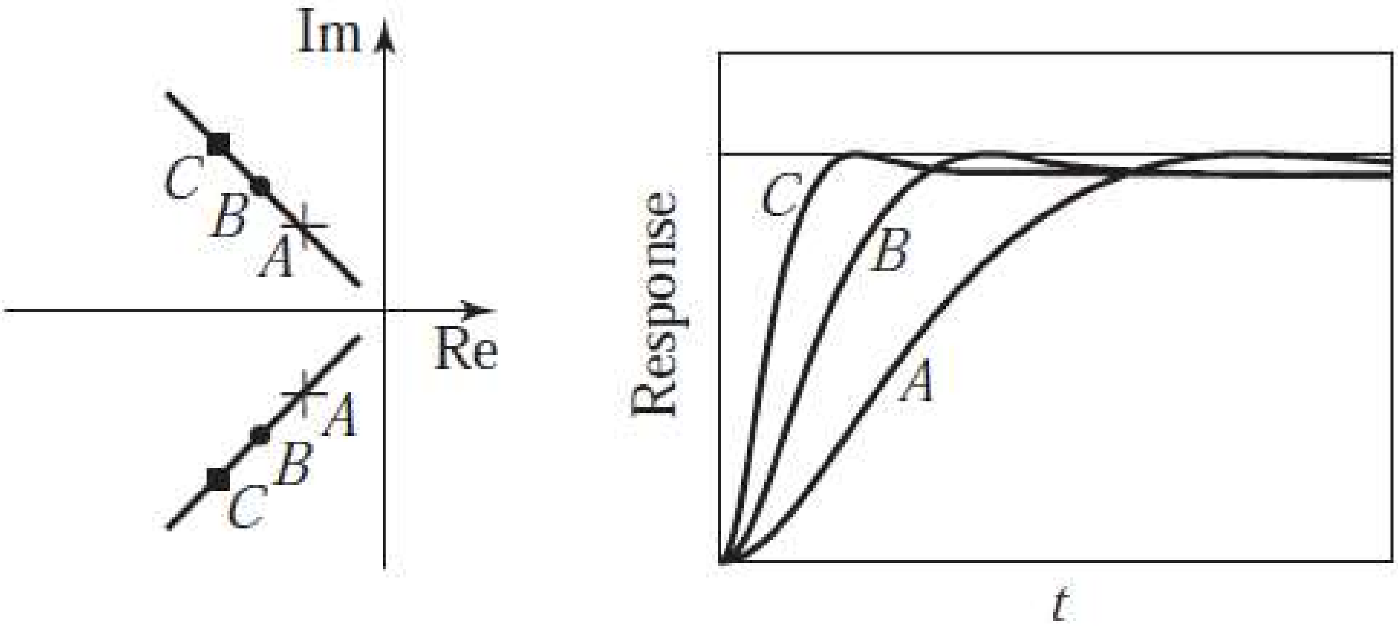
Root location and response

Models A and B have the same imaginary part, the same period, and the same peak time.

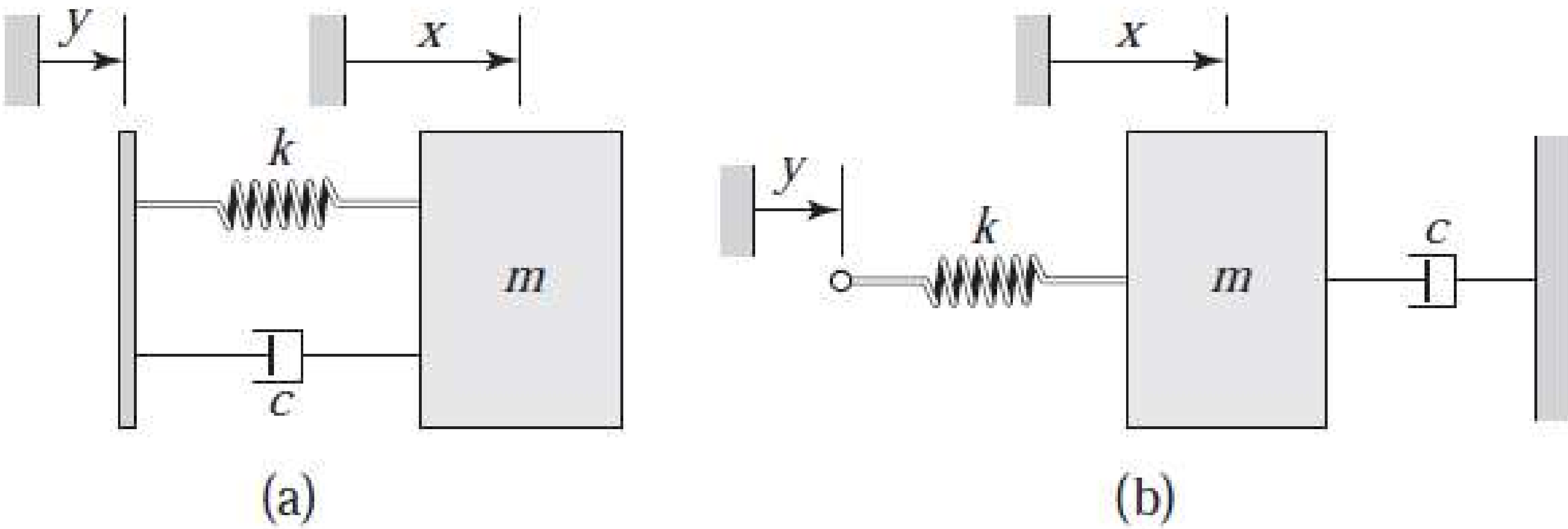


Root location and response

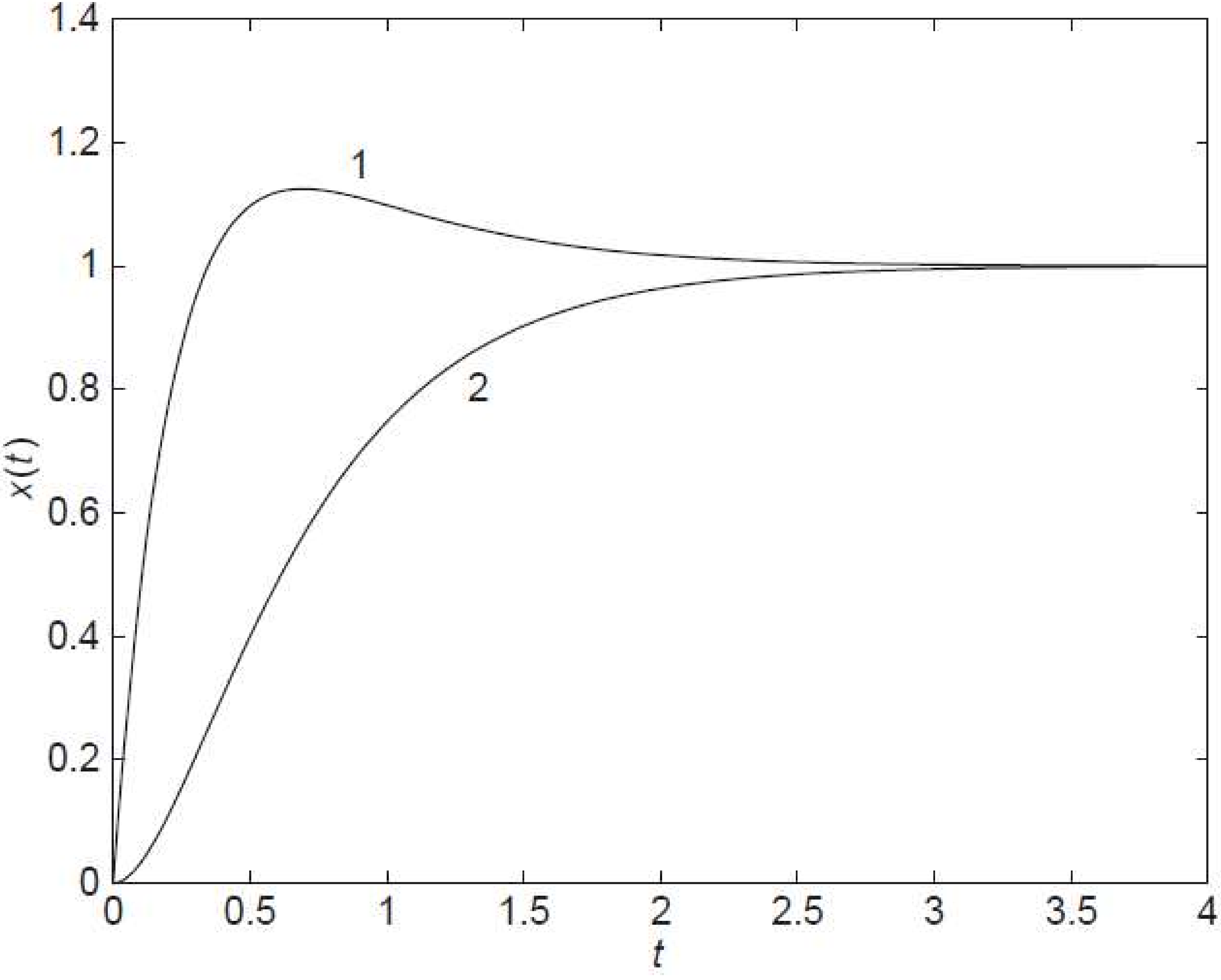
Models A, B, and C have the same damping ratio and the same overshoot.



Numerator dynamics and second order system response



- m=1;c=6;k=8



1.4
1.2
1
0.8
0.6
0.4
0.2
0
x(t)
0
0.5
1
1.5
2
2.5
3
3.5
4
t
1
2

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9.5 Introduction to block diagrams

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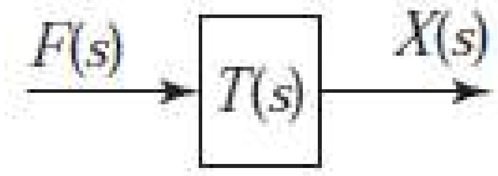
What is the block diagram

- The transfer function of a model can be used to construct a visual representation of the dynamics of the model.

  → Block diagram



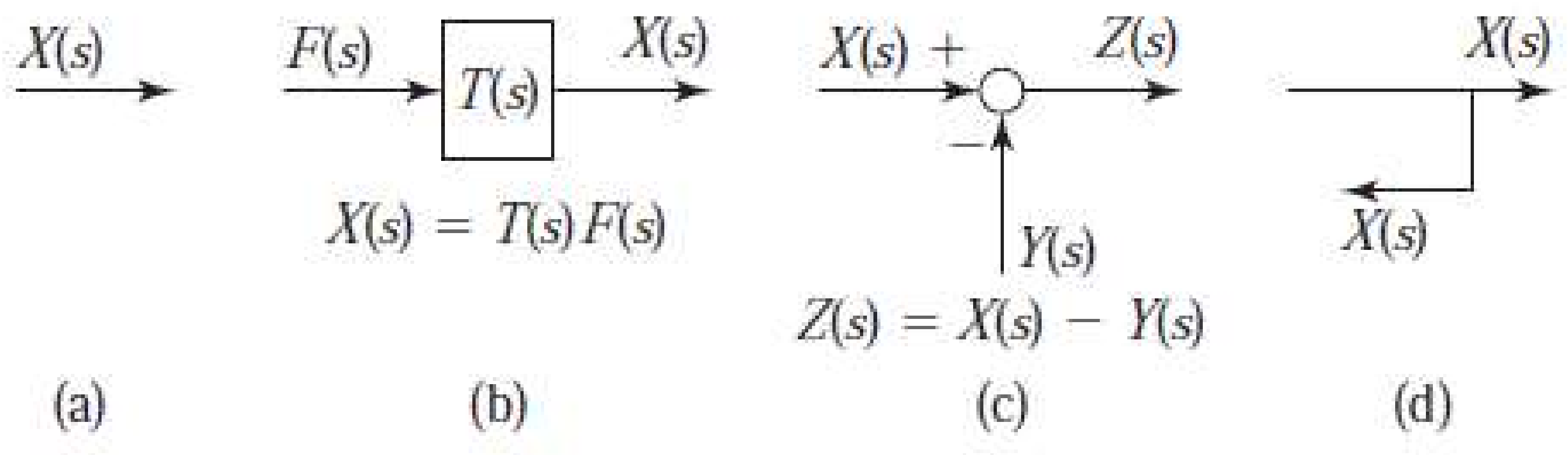
$$X(s) = T(s)F(s)$$

  It can describe:

- interaction of the components.
- Cause and effect relationship.
- Can be used to obtain transfer function.

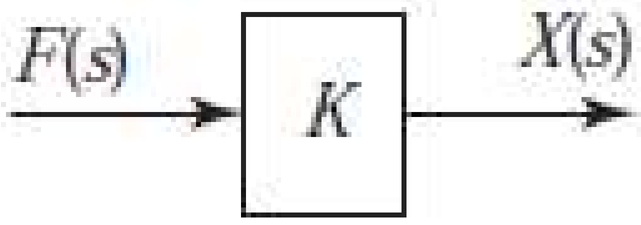
Block diagram symbols

- Four basic symbols



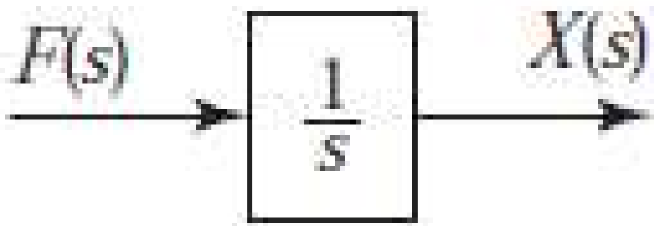
Simple examples

- Multiplier or gain



(a)

- Integrator



(b)

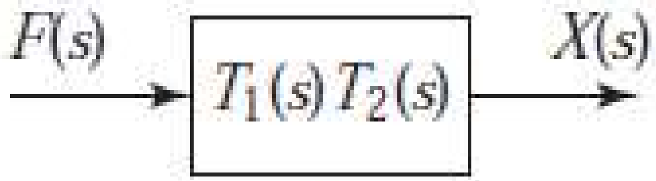
Equivalent block diagram

- Diagram of the right equation?

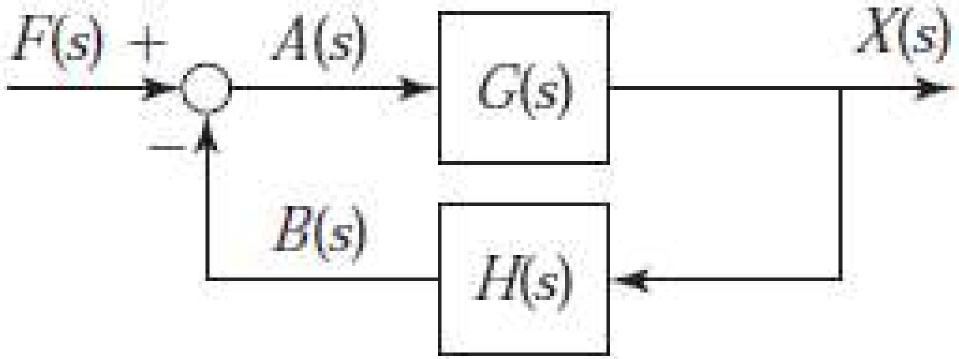
$$\dot{x} + 7x = f(t).$$

Serials elements and feedback loops

(a)

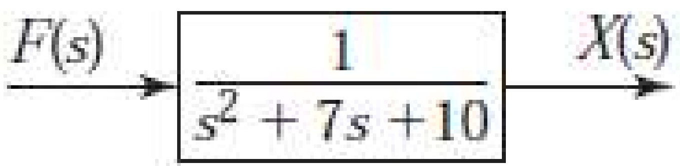


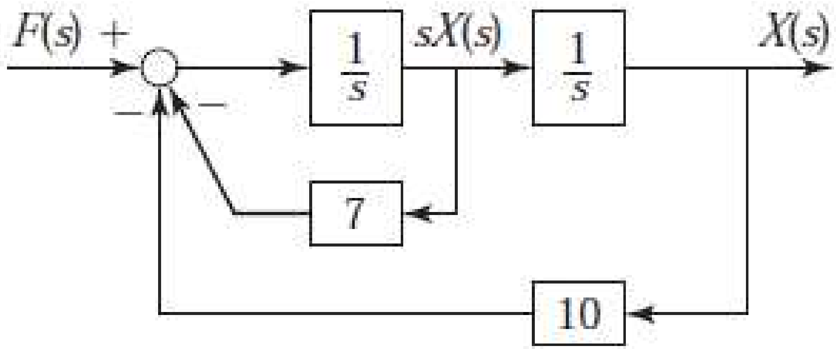
(b)

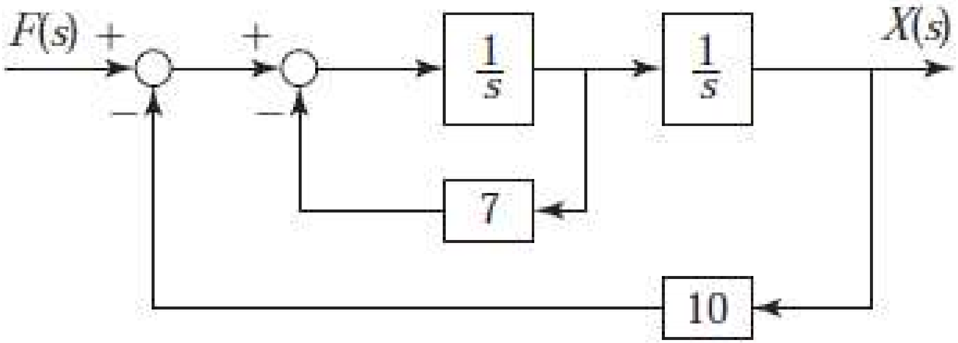


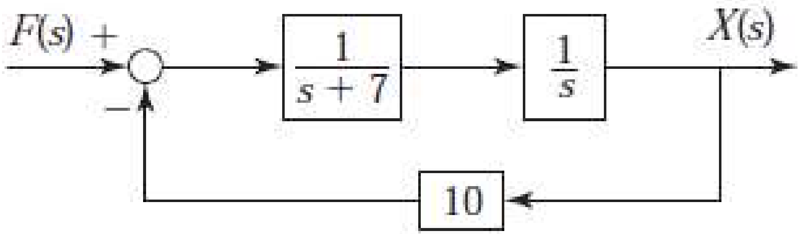
Rearranging block diagrams

$$\ddot{x} + 7\dot{x} + 10x = f(t)$$





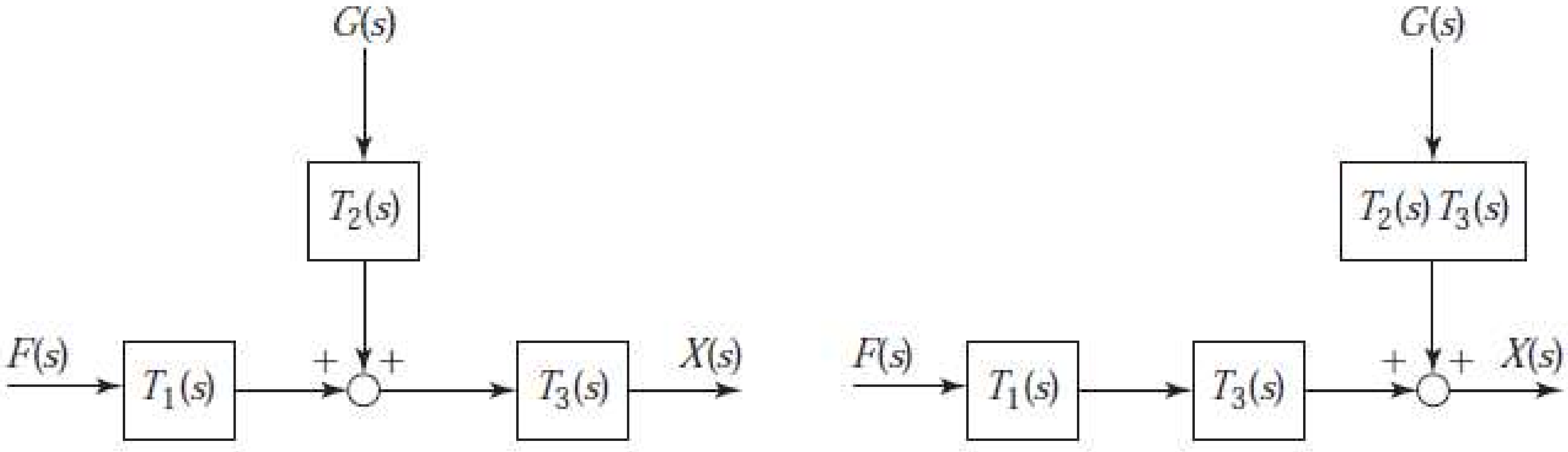




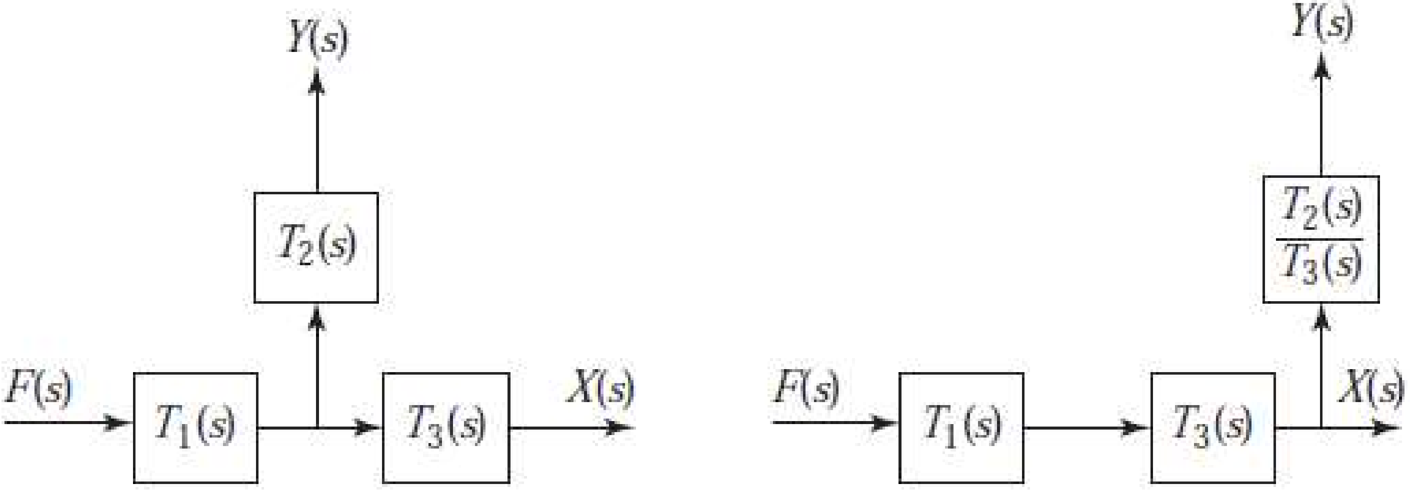
Important points

- Block diagram is not unique for a given equation. It depends on the requirements
- The form of block diagram depends on how the equation is arranged.
- Useful procedure: First solve the highest derivative of the dependent variable; the terms on the right side of the resulting equation represent the input to an integrator block.

Equivalent block diagram



(a)



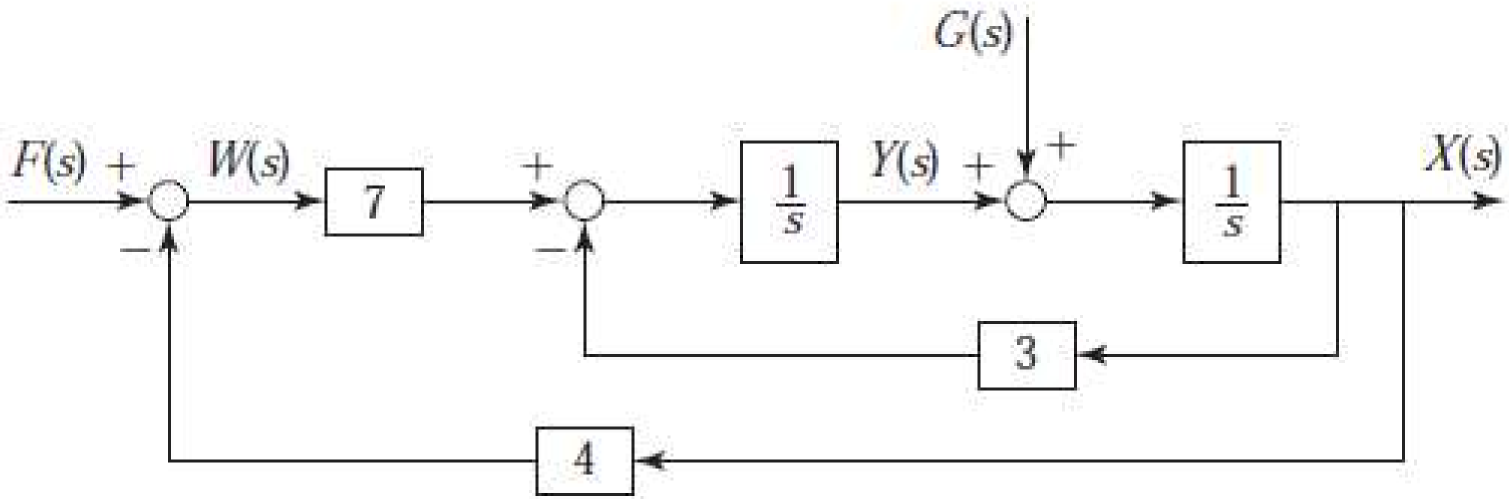
(b)

More differential equations

$$\dot{x} = -3y + f(t) \qquad \dot{y} = -5y + 4x + g(t)$$

Example 1

- Determine the model for output x



Example 2

- Derive the expression for C(s), E(s) and M(s) in terms of R(s) and D(s)